Transformations

In geometry we use input/output process when we determine how shapes are altered or moved. Geometric objects can be moved in the coordinate plane using a coordinate rule. These rules can alter the shape in many different ways. Some rules will translate the shape, some will rotate or reflect the shape, some will stretch or distort the shape, some will increase the size of the shape, etc… lots of different things can happen. Look at the example below:

Input **Output**

T(x,y) -------> (x + 3, y - 6)

Pre-Image **Image**

So rule T will move or map all points (x,y) by adding 3 to the x value of each point, and subtracting 6 from each y value of each point

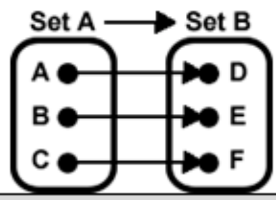
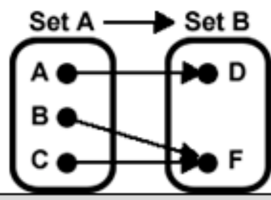
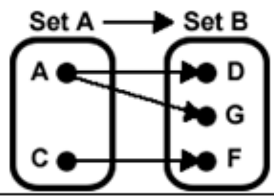
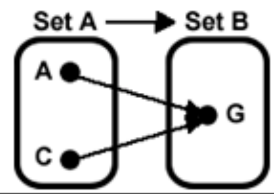
The name of the rule is a capital letter before the (x,y). The point (x,y) represents that **all points** in the plane will be effected by this rule. The arrow represents the geometric term of map or mapping, which is a geometric way of saying moves. Finally the coordinate description after the mapping arrow symbol represents the change produced by the rule. Can you see the connection to functions? You will input values into the coordinate rule and it will output new values, just as function did. A change is that we refer to the **input value** as the **pre-image**, the original location of the point, and the **output value** as the **image**, the new location of the point. **To help clarify the difference between a pre-image and an image, we use special notation. If the pre-image is point A, then its image will be A' (said A prime). The prime notation tells us that it is an image.**

1. Given coordinate rule **T(x, y) ⟶ (x + 3, y – 6) determine the image of A(-1, 5)?** **Pre-image Image**	**2. Given coordinate rule** **G(x, y) ⟶ (2x + 3, y + 1) determine the image of B(5, 2)?** **Pre-image Image**

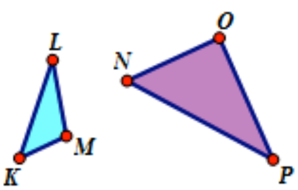
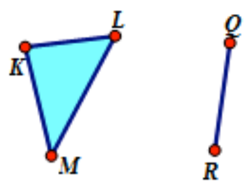
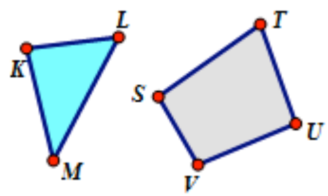
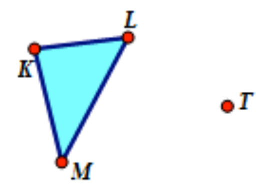
Working backwards is performing the inverse operation. + ↔ - and x ↔ ÷

3. Given coordinate rule **T(x, y) ⟶ (x + 3, y – 6) determine the pre-image of C\`(3, 2)?** **Pre-image Image**	**4. Given coordinate rule** **T(x, y) ⟶ (x + 3, y – 6) determine the pre-image of D\`(-4, 11)?** **Pre-image Image**

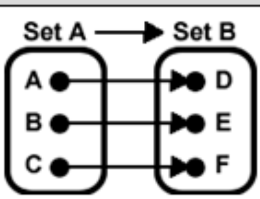
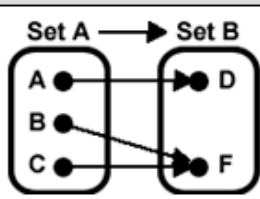
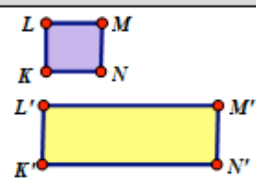
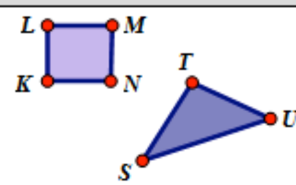
Mapping -- The word mapping is used in geometry as the word function is used in algebra. While a mapping is a correspondence between sets of points, a function is a correspondence between sets of numbers. **Function and Mapping mean the same thing but just in two different contexts, in Algebra and in Geometry.** You can not repeat the first value(x), but you can repeat the second value(y)

			
FUNCTION	**FUNCTION**	**NOT A FUNCTION**	**FUNCTION**
Each value set A has exactly one value in set B.	Each value set A has exactly one value in set B.	The A value has TWO values in set B.	Each value set A has exactly one value in set B.

A correspondence between the pre-image and image is a **MAPPING** IF AND ONLY IF each member of the pre-image corresponds to one and only one member of the image.

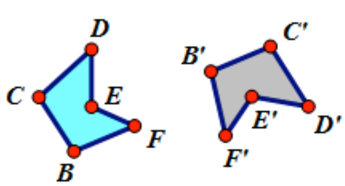
			
#1 MAPPING	**#2 MAPPING**	**#3 NOT A MAPPING**	**#4 MAPPING**
If the pre-image is ΔKLM, K maps to N L maps to O M maps to P	If the pre-image is ΔKLM, M maps to R K maps to Q L maps to Q	If the pre-image is ΔKLM, K maps to S L maps to T **M maps to U** **M maps to V**	If the pre-image is ΔKLM, K maps to T L maps to T M maps to T

In geometry, when you same the same number of points in the pre-image as in the image, it is called a TRANSFORMATION A transformation guarantees that if our pre-image has three points, then our image will also have three points.

ONE TO ONE FUNCTION	NOT ONE TO ONE FUNCTION	A TRANSFORMATION	NOT A TRANSFORMATION
			

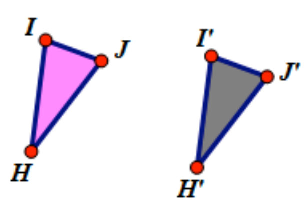
An **ISOMETRIC TRANSFORMATION (RIGID MOTION)** is a transformation that preserves the distances and/or angles between the pre-image and image.

Example #1



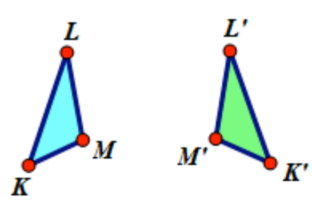
Rotate (Turn) – Example #1

Example #2



Translate (Slide) – Example #2

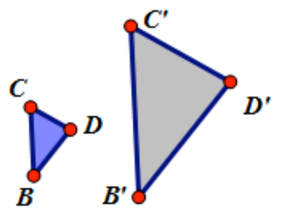
Example #3



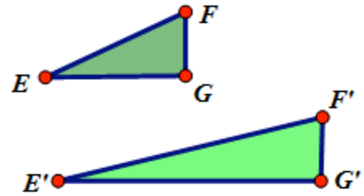
Reflection (Flip) - Example #3

A **NON-ISOMETRIC TRANSFORMATION (NON-RIGID MOTION)** is a transformation that does not preserve the distances and angles between the pre-image and image.

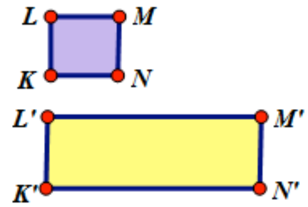
Example #1



Example #2



Example #3



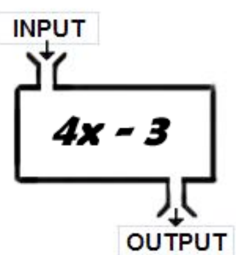
1. - 3. Use coordinate rule, T (x,y) ------ > (x + 5, y – 1) to determine the missing coordinates.

1. A (-4,5) A ' (____ , ____)	2. B (____ , ____) B' (-2,6)	3. C $(-6, \frac{4}{5})$ C ' (____ , ____)

4. – 6. Use coordinate rule, G (x,y) ------ > (x^2, 2y), to determine the missing coordinates.

4. a) A (-4,5) A ' (____ , ____)	5. B (-2,6) B' (____ , ____)	6. C (____ , ____) C' (16,-8)

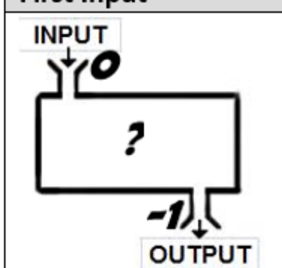
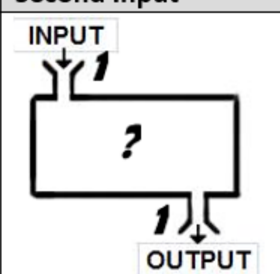
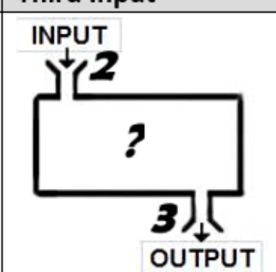
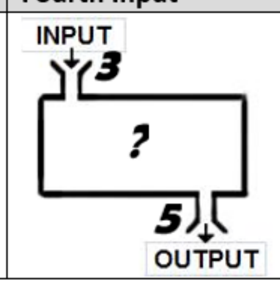
7. Using the function machine to determine the missing output or input values.



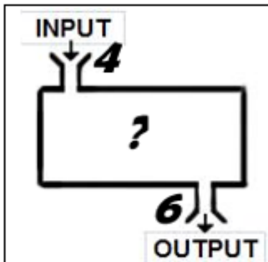
a) Input = -6 b) Input = $\frac{-2}{5}$ c) Input = ________ d) Input = ________

Output = _____ Output = ______ Output = -47 Output = 21

8. Determine the rule of the function machine.

First Input	Second Input	Third Input	Fourth Input	What is the rule for this function machine?
				

7. Determine three different function machine rules for the given input/output.

	a) Possible Rule #1	b) Possible Rule #2	c) Possible Rule #3
			

8. Use the given coordinate rules to solve missing coordinates.

a) T (x,y) ------ > (x, y + 7) A (-4,9 A' (_____ , _____) B (_____ , _____) B' (5,0)

b) S (x,y) ------ > (-y, x) A (-4,9) A' (_____ , _____) B (_____ , _____) B' (9,7)

c) F (x,y) ------ > (5x, 3y) A (-4,9) A' (_____ , _____) B (_____ , _____) B' (-5,12)

d) G (x,y) ------ > (-x, -3x) A (-4,9) A' (_____ , _____) B (_____ , _____) B' (-8,-24)

e) H (x,y) ------ > (2x - 1, y - 3) A (-4,9) A' (_____ , _____) B (_____ , _____) B' (31,15)

f) P (x,y) ------ > (x + 3, 2y) A (-4,9) A' (_____ , _____) B (_____ , _____) B' (5,8)

9. Write out the English translation for the following coordinate rule.

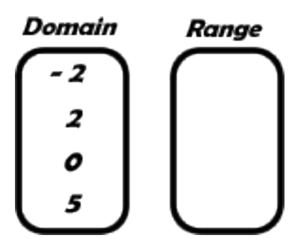
F (x,y) ------ > (x - 4, 2y) Coordinate Rule F ____

10. Complete the Analogy. Input IS TO Output AS Pre-Image IS TO _____________.

1. Given $f(x) = x^2$

a) find f(-2) = _________ b) find f(2) = _________ c) f(0) = _________ d) f(5) = _________

e) Use the data in 1a-d to complete the diagram to the right.



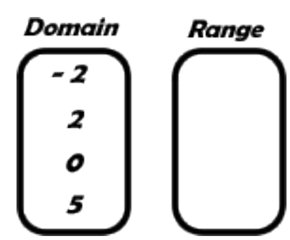
f) is $f(x) = x^2$ a function? YES OR NO

g) is $f(x) = x^2$ a one to one function? YES OR NO

iven $f(x) = -5x + 6$

d f(-2) = _________ b) find f(2) = _________ c) f(0) = _________ d) f(5) = _________

e) Use the data in 2a-d to complete the diagram to the right.

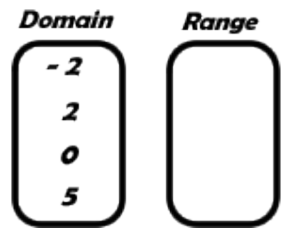


f) is $f(x) = -5x + 6$ a function? YES OR NO

g) is $f(x) = -5x + 6$ a one to one function? YES OR NO ven $f(x)=|x-1|$

d f(-2) = __________ b) find f(2) = __________ c) f(0) = __________ d) f(5) = __________

e) Use the data in 3a-d to complete the diagram to the right.

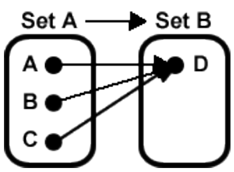


f) is $f(x)=|x-1|$ a function? YES OR NO

g) is $f(x)=|x-1|$ a one to one function? YES OR NO

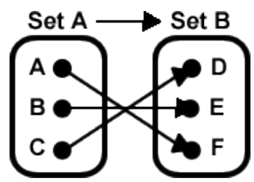
4. Determine whether the following are functions or not.

4.



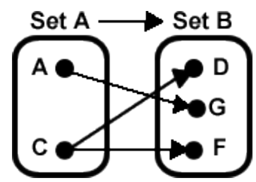
Function? YES OR NO

5.



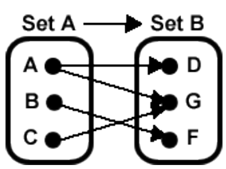
Function? YES OR NO

6.



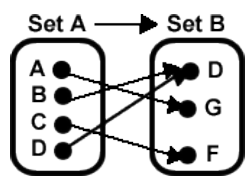
Function? YES OR NO

7.



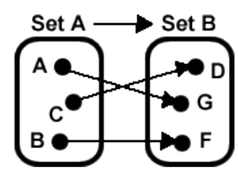
Function? YES OR NO

8.



Function? YES OR NO

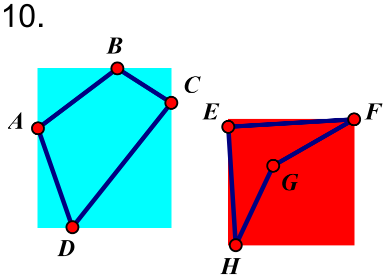
9.



Function? YES OR NO

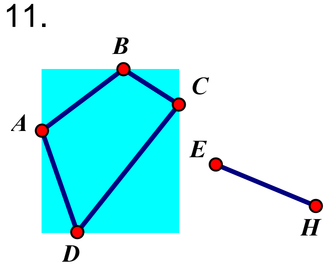
10. - 15. Given that the pre-image is Quadrilateral ABCD, determine which of the following could be classified as a mapping of the plane.

10.



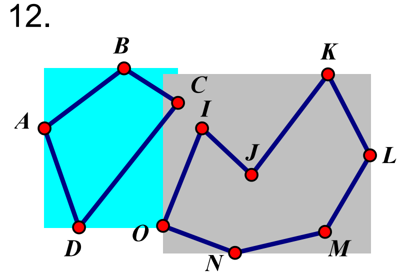
Function? YES OR NO

11.



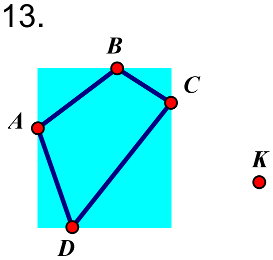
Function? YES OR NO

12.



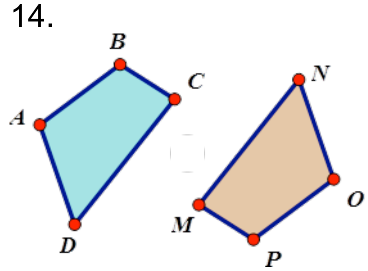
Function? YES OR NO

13.



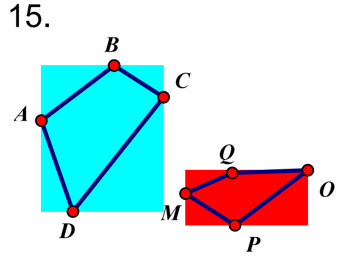
Function? YES OR NO

14.



Function? YES OR NO

15.



Function? YES OR NO

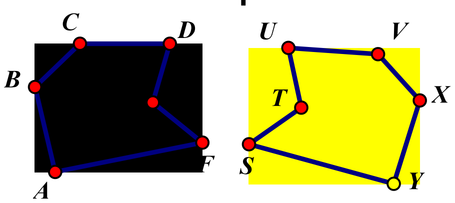
16. Which of the problems in question 5, would be classified as TRANSFORMATIONS?

____

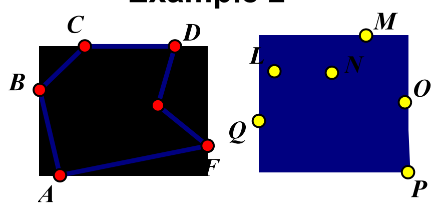
17. Transformations are a specific type of mapping. What makes them special from the general process of mapping?

18. Jeff is given a question on a test about transformations. He is given two examples both with pre-image hexagon ABCDE. The question asks if the two shapes are a transformation or not. On the first one he said **Yes** they are transformations because they are identical but in a different location and on the second one he said **No** that it was not a transformation because they we different shapes. **Is he correct? Explain why you agree or disagree?**

Example 1

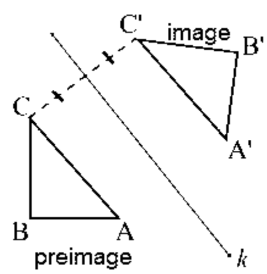


Example 2



Line of Reflections

The line of reflection is the perpendicular bisector of the segment joining every point and its image.



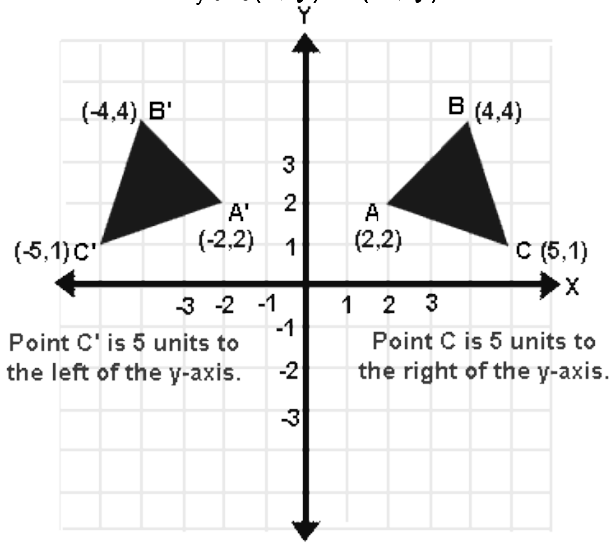
Remember that a reflection is a **flip**. Under a reflection, the figure does not change size.
A line reflection creates a figure that is congruent to the original figure and is called an **isometry** (a transformation that preserves length). Since naming (lettering) the figure in a reflection requires changing the order of the letters (such as from clockwise to counterclockwise), a reflection is more specifically called a non-direct or **opposite isometry**.

Under a Line reflection, distance, angle measurement, collinearity and midpoint are preserved

Line Reflections in the Coordinate Plane

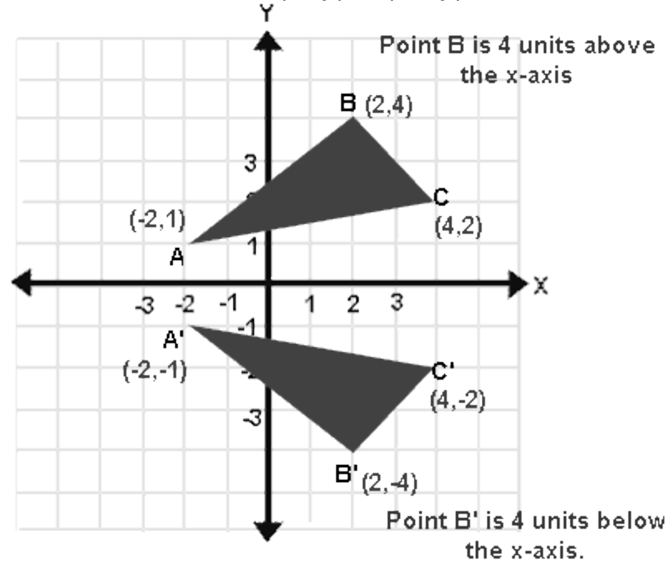
Reflection in the y-axis → P(x, y) is P\`(-x, y)

$r_{y\text{-axis}}(x, y) = (-x, y)$

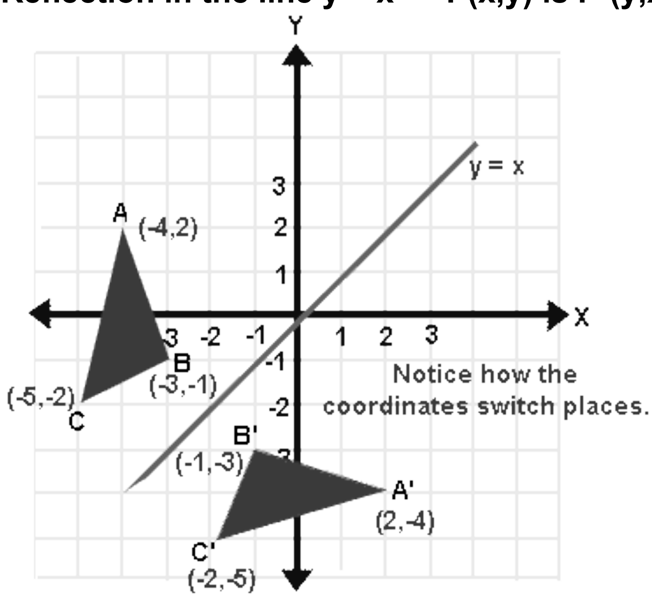


Reflection in the x-axis → P(x, y) is P\`(x, -y)

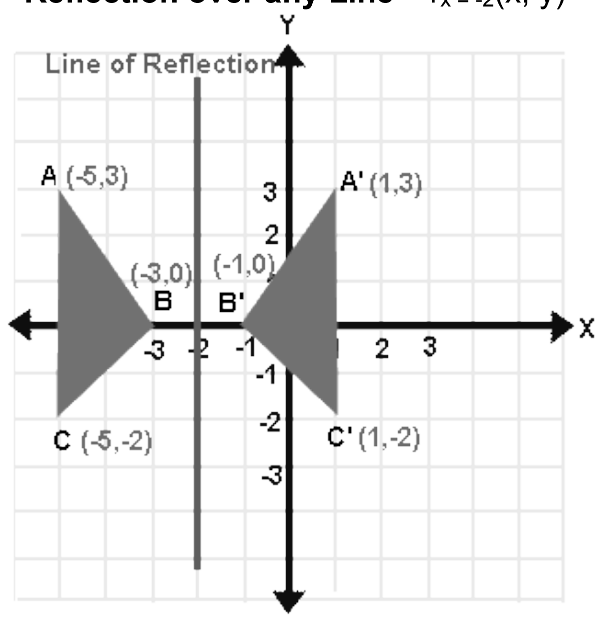
$r_{x\text{-axis}}(x, y) = (x, -y)$



Reflection in the line y = x → P(x,y) is P\`(y,x)



Reflection over any Line $r_{x=-2}(x, y)$

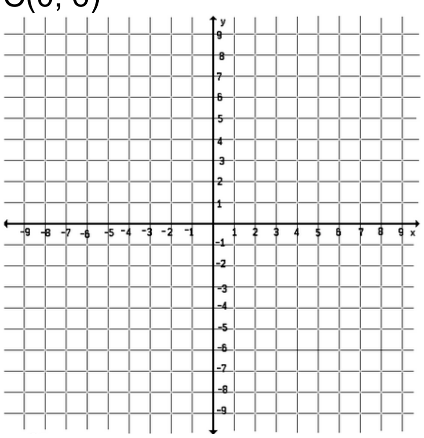


1. Under a reflection in the x-axis, the image of (x, y) ________
2. Under a reflection in the y-axis, the image of (x, y) ________
3. Under a reflection in the line y = x, the image of (x, y) _______

Find the image of the point under a reflection in the x-axis			
4. (5, 7)	5. (6, 2)	6. (-1, -4)	7. (3, 0)
Find the image of the point under a reflection in the y-axis			
8. (5, 7)	9. (-4, 10)	10. (0, 6)	11. (-1, -6)
Find the image of the point under a reflection in the line y = x			
12. (5, 7)	13. (-3, 8)	14. (0, -2)	15. (6, 6)

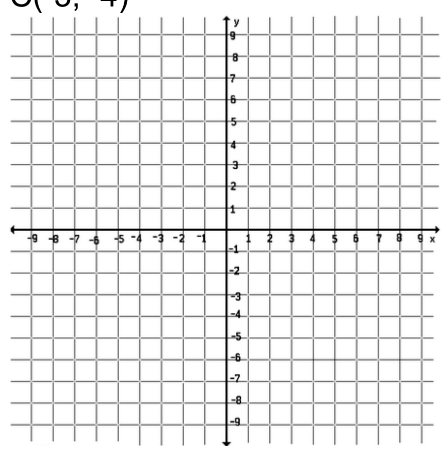
16. The vertices of ΔABC are A(3, 0), B(3, 6) and C(0, 6). Find the coordinates of the images of the vertices under the given reflection:

Given	the x-axis	the y-axis	the line y = x
A(3, 0)			
B(3, 6)			
C(0, 6)			



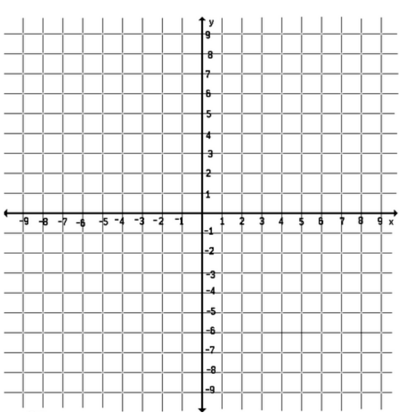
17. The vertices of ΔABC are A(1, 4), B(3, 0) and C(-3, -4). Find the coordinates of the images of the vertices under the given reflection:

Given	the x-axis	the y-axis	the line y = x
A(1, 4)			
B(3, 0)			
C(-3, -4)			

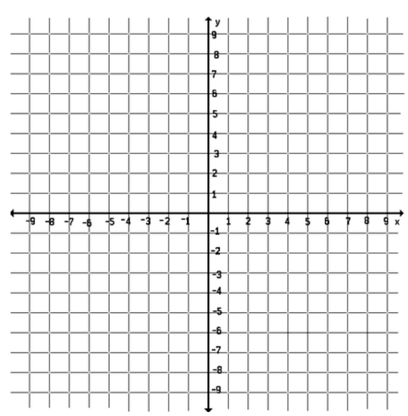


Using the given coordinates, draw ΔABC and ΔA`B`C` on one set of axes and then find the equation of the line of reflection.

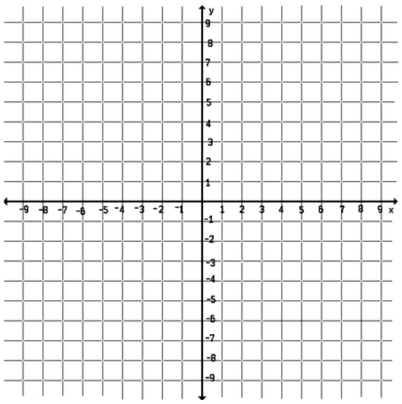
18. ΔABC: A(2, 4), B(2, 1) and C(-1, 1)
ΔA`B`C`: A`(4, 4), B`(4, 1) and C`(7, 1)



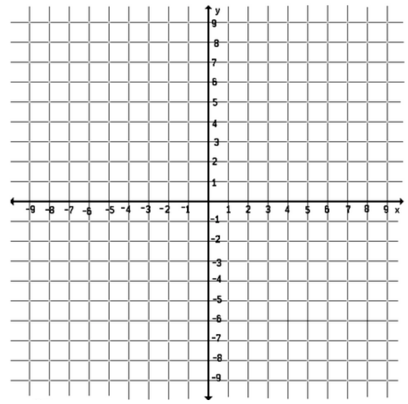
19. ΔABC: A(1, 3), B(2, 5) and C(5, 3)
ΔA`B`C`: A`(1, 1), B`(2, -1) and C`(5, 1)



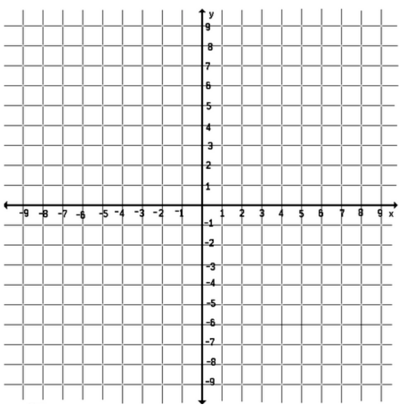
20. ΔABC: A(1, 4), B(2, 1) and C(4, 2)
ΔA`B`C`: A`(-5, 4), B`(-6, 1) and C(`-8, 2)



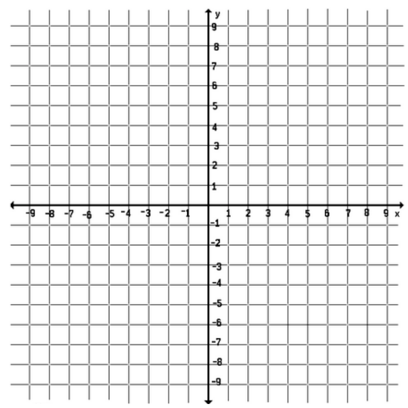
21. ΔABC: A(4, 2), B(6, 2) and C(2, -1)
ΔA`B`C`: A`(2, 4), B`(2, 6) and C`(-1, 2)



22. ΔABC: A(-1, 0), B(0, 2) and C(4, 1)
ΔA`B`C`: A`(-1, 8), B`(0, 6) and C`(4, 7)



23. ΔABC: A(2, -1), B(4, 2) and C(-1, 2)
ΔA`B`C`: A`(1, -2), B`(-2, -4) and C`(-2, 1)



(1) REFLECTION OVER THE Y AXIS

PRE-IMAGE LOCATION	REFLECTION	IMAGE LOCATION
a) Place ΔDEF on the coordinate grid at D(0,6), E(3,6) and F(0,2)	$R_{y\ axis}$	D'(___, ___), E'(___, ___), & F'(___, ___)
b) Place ΔABC on the coordinate grid at A(3,0), B(5,-5) and C(1,-5)	$R_{y\ axis}$	A'(___, ___), B'(___, ___), & C'(___, ___)
c) Place ΔMNP on the coordinate grid at M(0,-1), N(7,-4) and P(2,-4)	$R_{y\ axis}$	M'(___, ___), N'(___, ___), & P'(___, ___)

Create a transformation rule for reflection over the y axis. $R_{y\ axis}(x, y) = (\_\_\_\_\_\_, \_\_\_\_\_\_)$

2) REFLECTION OVER THE X AXIS

PRE-IMAGE LOCATION	REFLECTION	IMAGE LOCATION
a) Place ΔDEF on the coordinate grid at D(-7,-2), E(-4,-2) and F(-7,-6)	$R_{x\ axis}$	D'(___, ___), E'(___, ___), & F'(___, ___)
b) Place ΔABC on the coordinate grid at A(6,-1), B(8,-6) and C(4,-6)	$R_{x\ axis}$	A'(___, ___), B'(___, ___), & C'(___, ___)
c) Place ΔMNP on the coordinate grid at M(-1,-2), N(6,-5) and P(1,-5)	$R_{x\ axis}$	M'(___, ___), N'(___, ___), & P'(___, ___)

Create a transformation rule for reflection over the x axis. $R_{x\ axis}(x, y) = (\_\_\_\_\_\_, \_\_\_\_\_\_)$

(3) REFLECTION OVER THE Y = 1X LINE

PRE-IMAGE LOCATION	REFLECTION	IMAGE LOCATION
a) Place ΔDEF on the coordinate grid at D(0,6), E(3,6) and F(0,2)	$R_{y=x}$	D'(___, ___), E'(___, ___), & F'(___, ___)
b) Place ΔABC on the coordinate grid at A(3,0), B(5,-5) and C(1,-5)	$R_{y=x}$	A'(___, ___), B'(___, ___), & C'(___, ___)
c) Place ΔMNP on the coordinate grid at M(0,-1), N(7,-4) and P(2,-4)	$R_{y=x}$	M'(___, ___), N'(___, ___), & P'(___, ___)

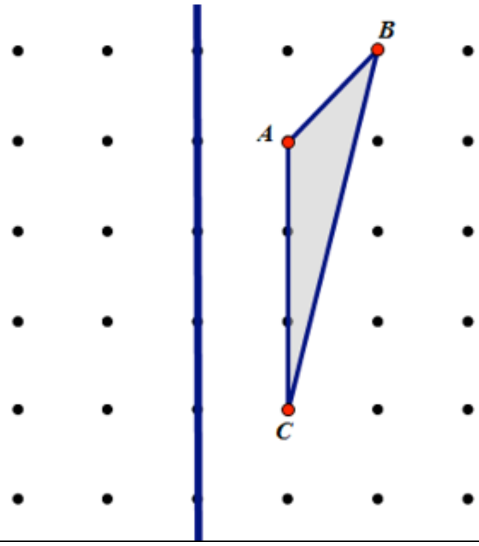
Create a transformation rule for reflection over the y = x line. $R_{y=x}(x, y) = (\_\_\_\_\_\_, \_\_\_\_\_\_)$

(4) REFLECTION OVER A VERTICAL LINE (x = c) (c is the variable representing all possible vertical lines)

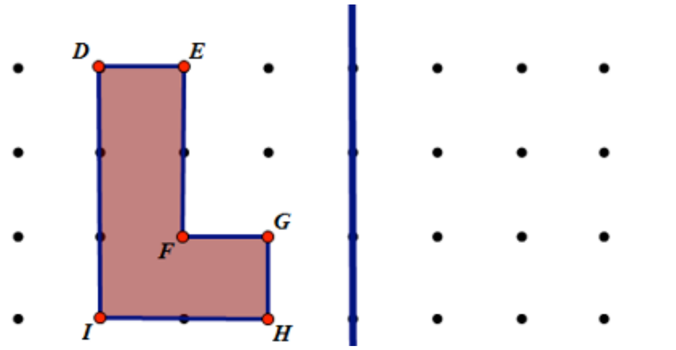
PRE-IMAGE LOCATION	REFLECTION	IMAGE LOCATION
a) Place ΔDEF on the coordinate grid at D(-4,5), E(-1,5) and F(-4,1)	$R_{x=0}$ (y axis)	D'(___, ___), E'(___, ___), & F'(___, ___)
b) Place ΔDEF on the coordinate grid at D(-4,5), E(-1,5) and F(-4,1)	$R_{x=1}$	D'(___, ___), E'(___, ___), & F'(___, ___)
c) Place ΔDEF on the coordinate grid at D(-4,5), E(-1,5) and F(-4,1)	$R_{x=2}$	D'(___, ___), E'(___, ___), & F'(___, ___)
d) Place ΔDEF on the coordinate grid at D(-4,5), E(-1,5) and F(-4,1)	$R_{x=3}$	D'(___, ___), E'(___, ___), & F'(___, ___)
e) Place ΔDEF on the coordinate grid at D(-4,5), E(-1,5) and F(-4,1)	$R_{x=4}$	D'(___, ___), E'(___, ___), & F'(___, ___)

1. – 10. Use the to reflect the following figures over their respective line m. Label the image.

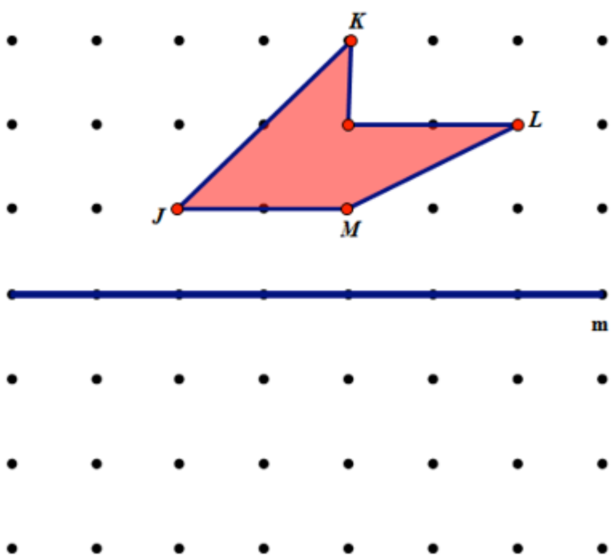
1.



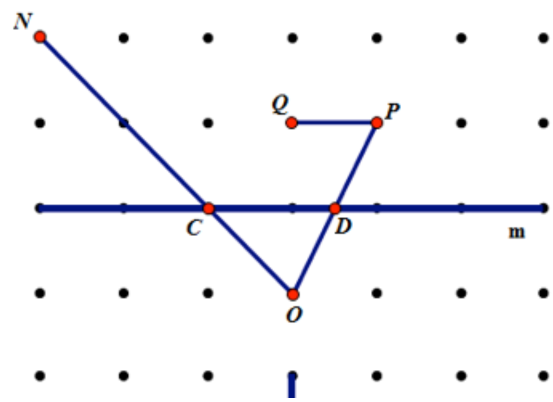
2.



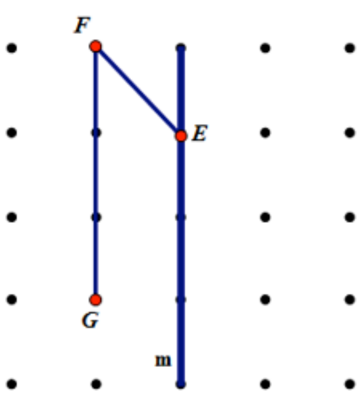
3.



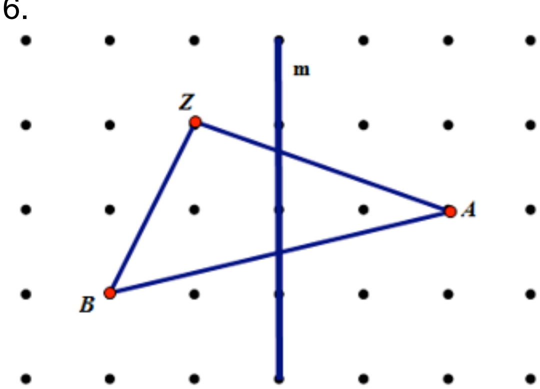
4.



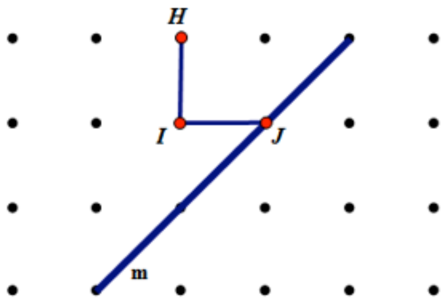
5.



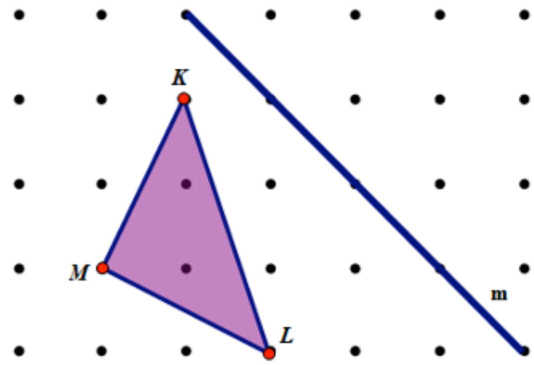
6.



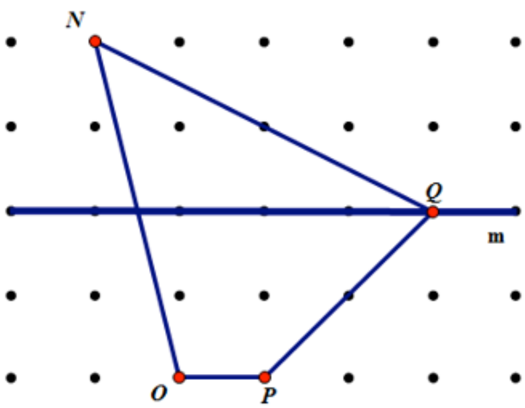
7.



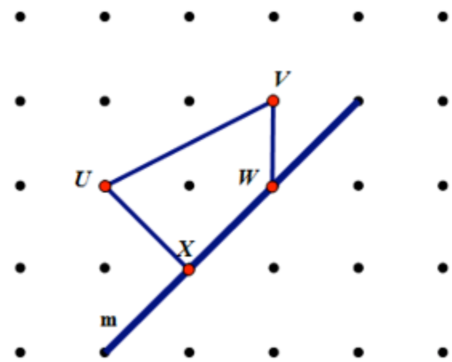
8.



9.

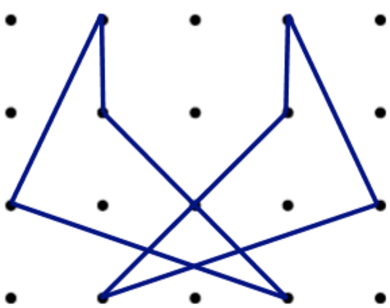


10.

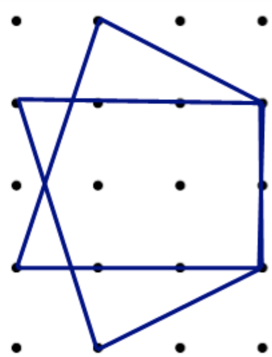


11. – 16. Determine the line of reflection for the following pre-image and images.

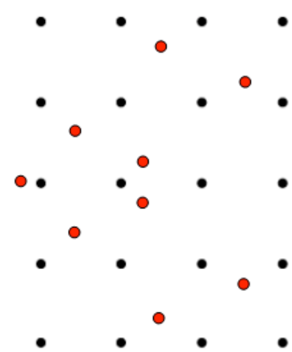
11.



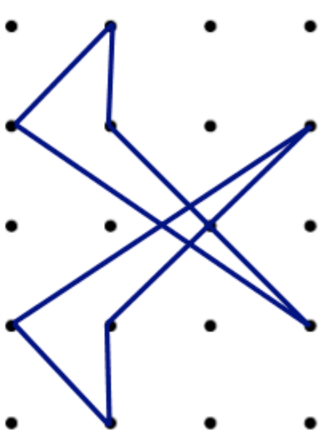
12.



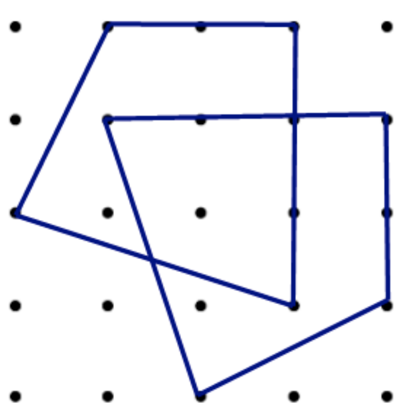
13.



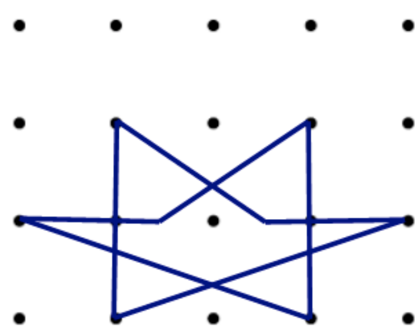
14.



15.



16.



3. Determine the pre-image coordinates, then reflect it, and determine the image coordinates.

a) A = (____ , ____) $R_{x\ axis}(A)$ A ‘ = (____ , ____)

b) B = (____ , ____) $R_{y\ axis}(B)$ B ‘ = (____ , ____)

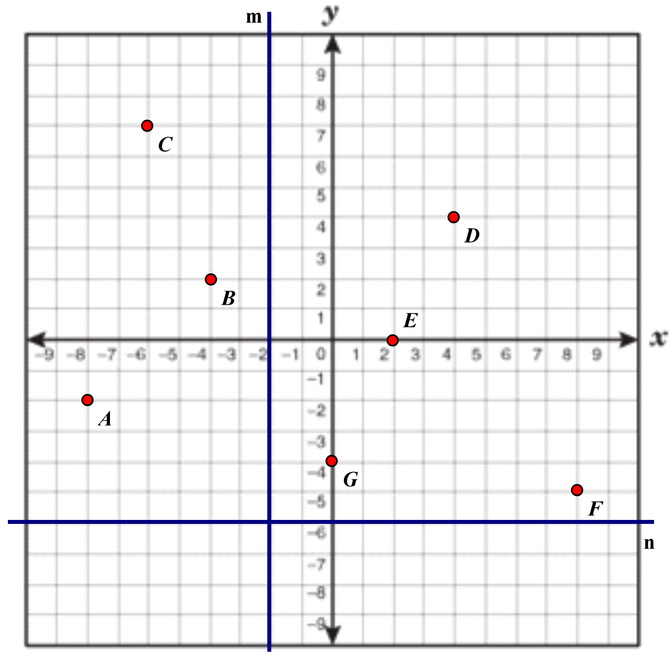
c) C = (____ , ____) $R_{m}(C)$ C ‘ = (____ , ____)

d) D = (____ , ____) $R_{x\ axis}(D)$ D ‘ = (____ , ____)

e) E = (____ , ____) $R_{x\ axis}(E)$ E ‘ = (____ , ____)

f) F = (____ , ____) $R_{n}(F)$ F ‘ = (____ , ____)

g) G = (____ , ____) $R_{y\ axis}(G)$ G ‘ = (____ , ____)



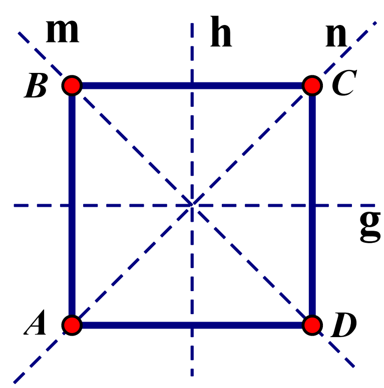
4. Determine the name of the point that meets the given conditions.

a) $R_{m}(A) =$ ________ b) $R_{h}(C) =$ ________

c) $R_{h}(D) =$ ________ d) $R_{g}($________$) =$ B

e) $R_{n}(D) =$ ________ f) $R_{n}(B) =$ ________

g) $R_{m}(D) =$ ________ h) $R_{m}($________$) =$ C



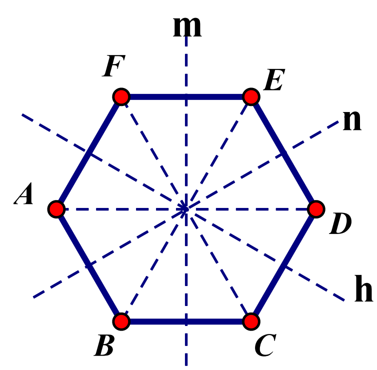
5. Determine the name of the point that meets the given conditions.

a) $R_{m}(A) =$ ________ b) $R_{\overline{FC}}(A) =$ ________

c) $R_{h}(B) =$ ________ d) $R_{\overline{AD}}($________$) =$ B

e) $R_{n}(D) =$ ________ f) $R_{n}(B) =$ ________

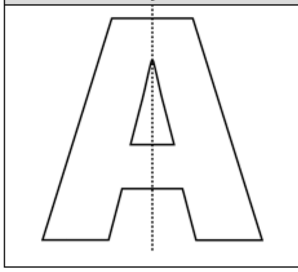
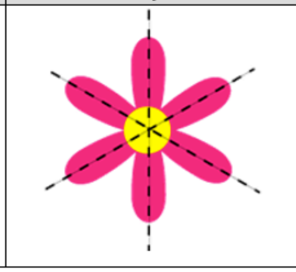
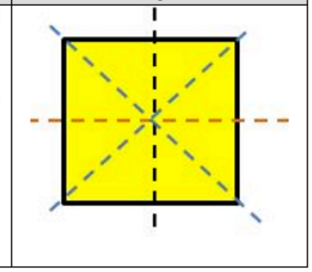
g) $R_{\overline{FC}}(D) =$ ________ h) $R_{\overline{BE}}($________$) =$ C



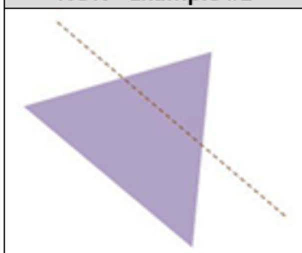
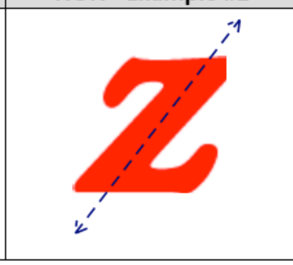
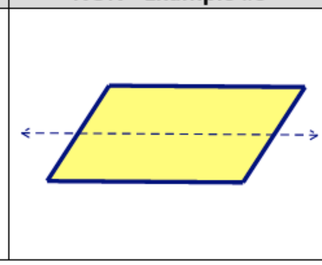
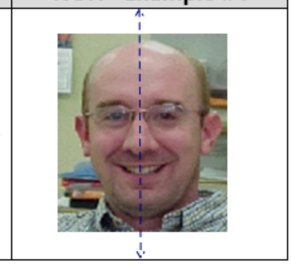
LINE SYMMETRY (or REFLECTIONAL SYMMETRY)

A line of symmetry divides a figure into halves that are the ***mirror images*** of each other.

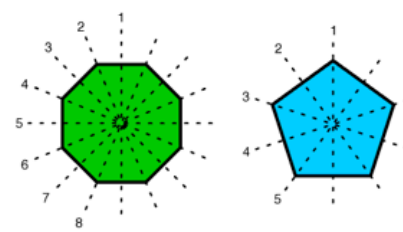
Below are some examples of figures that do have line symmetry. A shape can have more than one line symmetry.

Example #1	Example #2	Example #3	Example #4
			

Here are a few non-examples of shapes DO NOT have line symmetry.

NON - Example #1	NON - Example #2	NON - Example #3	NON - Example #4
			

The maximum lines of symmetry that a polygon can have are equal to its number of sides. The maximum is always found in the regular polygon, because all sides and all angles are congruent.



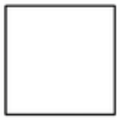
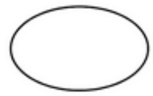
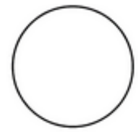
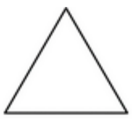
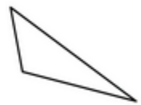
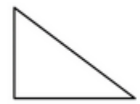
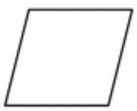
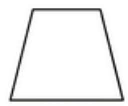
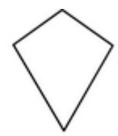
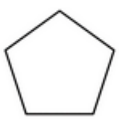
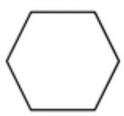
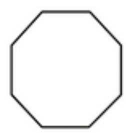
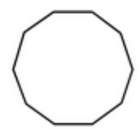
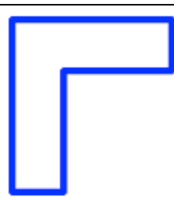
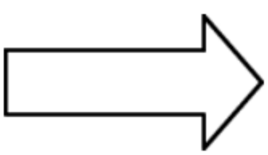
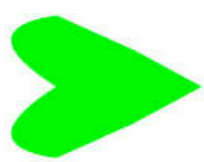
1. Below are the letters of the alphabet. Draw one or more lines of reflections for each of these letters, if they have one.

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
J	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**
S	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**	

2. **Tell which of the following words have line symmetry and if such symmetry exists, draw the line.**

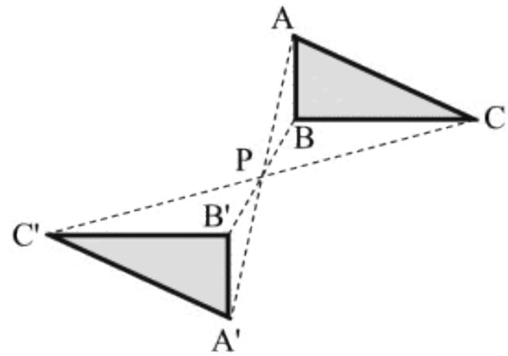
DAD	YOUTH	HIKED	CHECK	BOB
DEED	RADAR	CHOKED	AVA	TOT
AXION	MOM	CHO	HOE	DOB

3. **Draw all lines of symmetry in the geometric shapes, if there are none, leave alone:**

 **square**	 **rectangle**	 **eclipse**	 **circle**
 **equilateral triangle**	 **isosceles triangle**	 **scalene triangle**	 **right triangle**
 **rhombus**	 **parallelogram**	 **isosceles trapezoid**	 **kite**
 **pentagon**	 **hexagon**	 **octagon**	 **decagon**
			
			

Point Reflection

A **Point Reflection** creates a figure that is congruent to the original figure and is called an **isometry** (a transformation that preserves length). Since the orientation in a point reflection remains the same (such as counterclockwise seen in this diagram), a point reflection is more specifically called a **direct isometry.** **P** is the midpoint of each line AA`, BB` CC`



Under a Point of Reflection, distance, angle measurement, collinearity and midpoint are preserved.

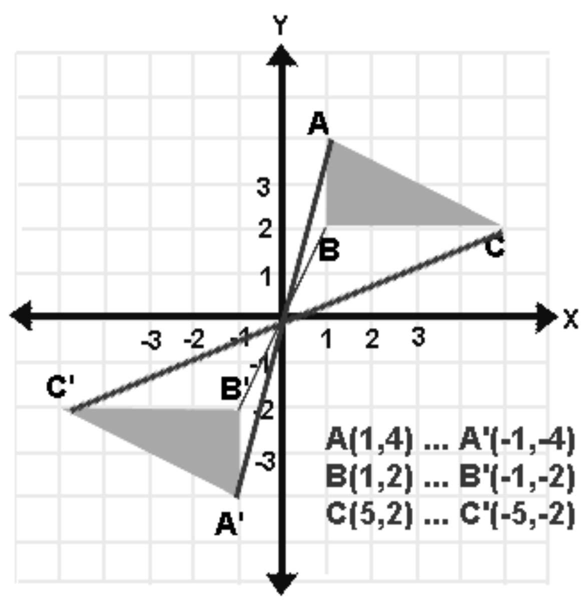
Point of Symmetry

In a point symmetry, the center point is a **midpoint** to every segment formed by joining a point to its image.

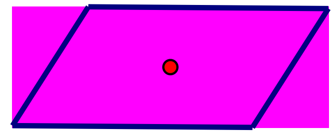
Point Reflection in the Coordinate Plane

Reflection over the Origin P(x, y) is P`(-x, -y)

$r_o (x, y) = (-x, -y)$



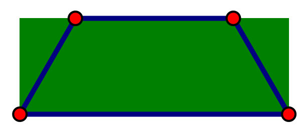
Here are some examples of shapes that have rotation symmetry.

Example #1	Example #2	Example #3	Example #4
			

ANGLE OF ROTATION - When a shape has rotational symmetry we sometimes want to know what the angle of rotational symmetry is. To determine this we determine the SMALLEST angle through which the figure can be rotated to coincide with itself. This number will always be a factor of 360. So in the examples above, example #1 would have angle of rotational symmetry of 120, example #2 would have an angle of 180, example #3 would have an angle of 120° and finally example #4 would have an angle of 180.

ORDER OF A ROTATION SYMMETRY -- The number of positions in which the object looks exactly the same is called the **order** of the symmetry. When determining order, the last rotation returns the object to its original position. **Order 1 implies no true rotational symmetry since a full 360 degree rotation was needed.** So in the four examples of rotation symmetry, example #1 has order 3, example #2 has order 2, example #3 has order 3, and example #4 has order 2.

Here are a few non-examples of shapes DO NOT have rotation symmetry.

NON - Example #1	NON - Example #2	NON - Example #3	NON - Example #4
 (the colors don't work)	 (numbers don't work)		

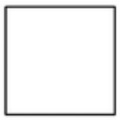
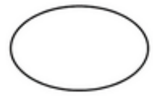
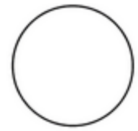
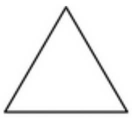
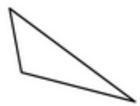
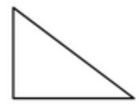
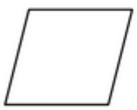
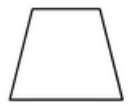
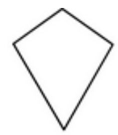
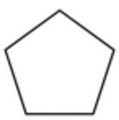
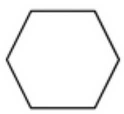
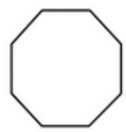
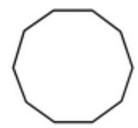
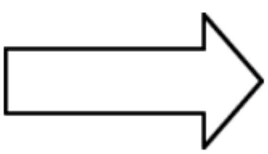
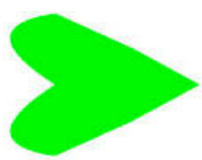
1. Draw a point of symmetry for each letter that has one.

A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
J	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**
S	**T**	**U**	**V**	**W**	**X**	**Y**	**Z**	

2. Draw a point of symmetry if the word has one.

SIS	WOW	NOON	ZOO	XOX
un	pod	CHOKED	SWIMS	OX

3. Draw points of symmetry in the geometric shapes, and give the angle of rotation and the order of rotation.

 **square**	 **rectangle**	 **eclipse**	 **circle**
 **equilateral triangle**	 **isosceles triangle**	 **scalene triangle**	 **right triangle**
 **rhombus**	 **parallelogram**	 **isosceles trapezoid**	 **kite**
 **pentagon**	 **hexagon**	 **octagon**	 **decagon**
			
			

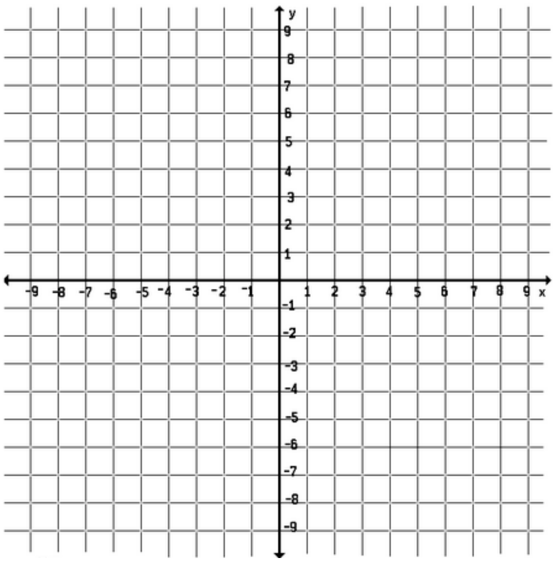
4. Under a reflection in the origin, the image of (x, y) is _______

5. Using the reflection through the origin, find the images of A(2, 1), B(4, 5) and C(-1, 3) and draw ∆ABC and ∆A`B`C` on the set of axis.

A(2, 1) →

B(4, 5) →

C(-1, 5) →



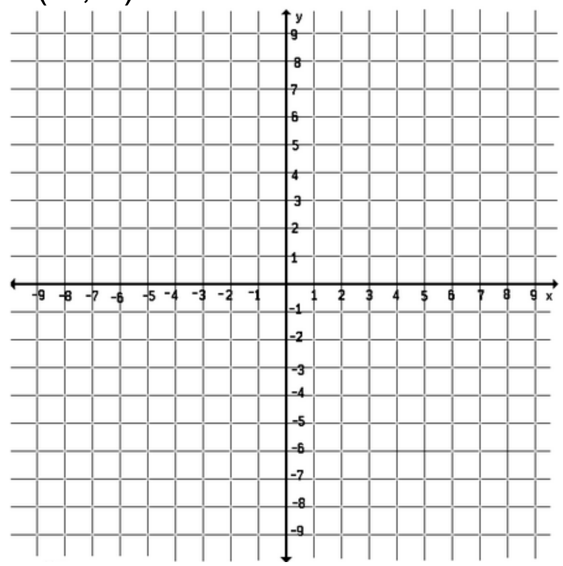
6. Using the reflection through the origin, find the images of A(-5, -2), B(-1, -2) and C(-1, 4), and D(-3, 3) and draw ABCD and A`B`C`D` on the set of axis.

A(-5, -2) →

B(-1, -2) →

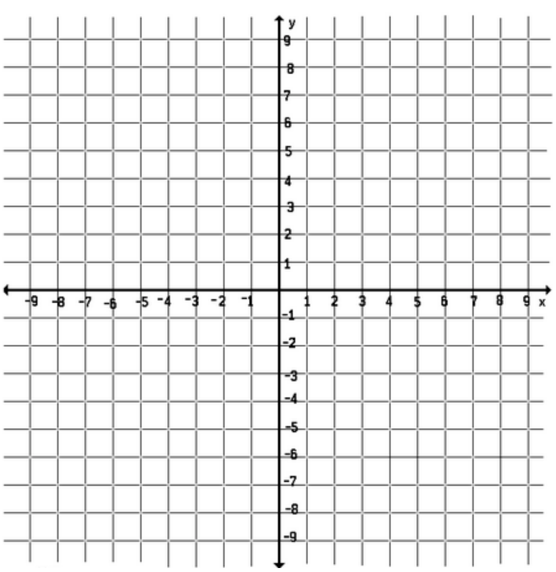
C(-1, 4) →

D(-3, 3) →

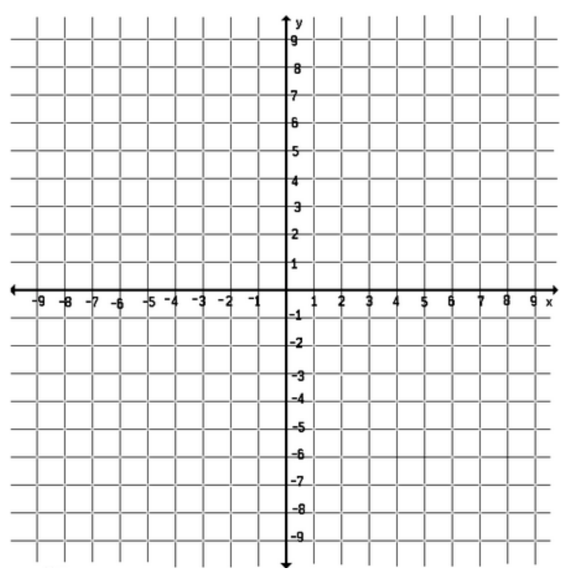


The image of ∆ABC under a point of reflection is ∆A`B`C`. Using the given coordinates, draw ∆ABC and ∆A`B`C` on one set of axis, then find the coordinates of the point of symmetry

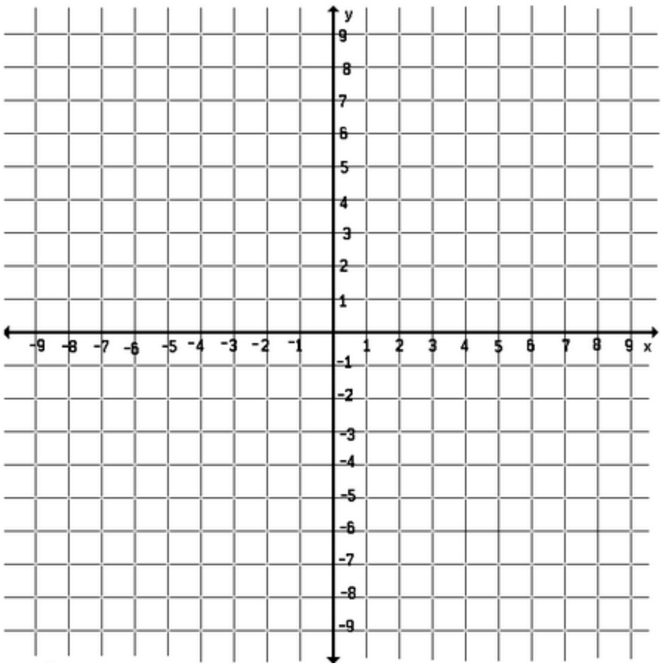
7. ∆ABC: A(2, 2), B(5, 2) and C(5, 4)
∆A`B`C`: A`(0, 2), B`(-3, 2) and C`(-3, 0)



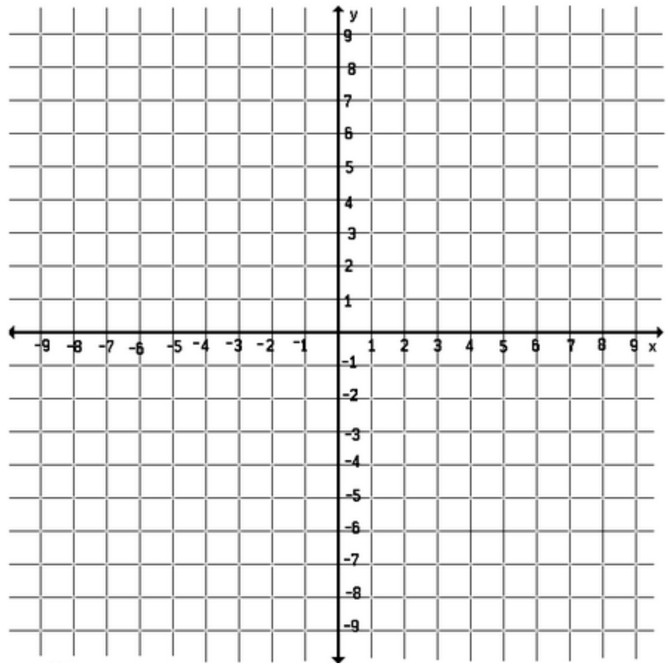
8. ∆ABC: A(2, 5), B(5, 6) and C(2, 3)
∆A`B`C`: A`(2, 1), B`(-1, 0) and C`(2, 3)



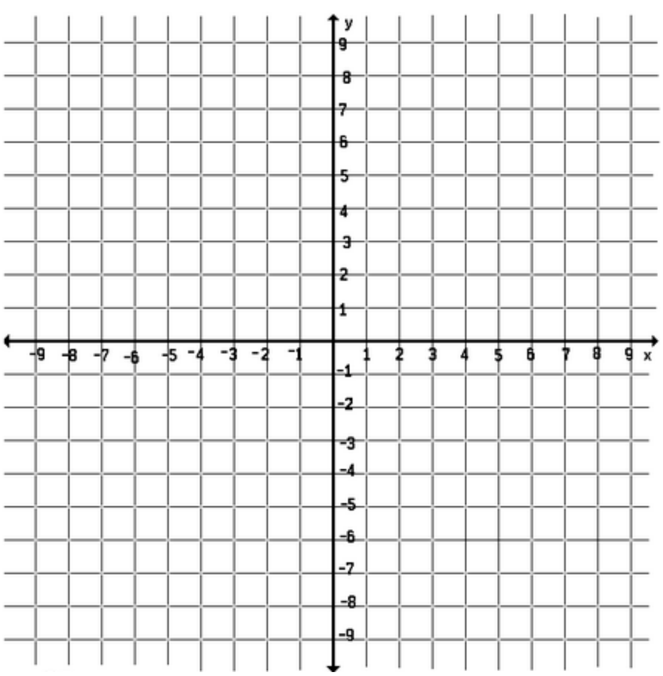
9. ∆ABC: A(3, 1), B(5, 5) and C(6, 3)
∆A`B`C`: A`(3, 7), B`(1, 3) and C`(0, 5)



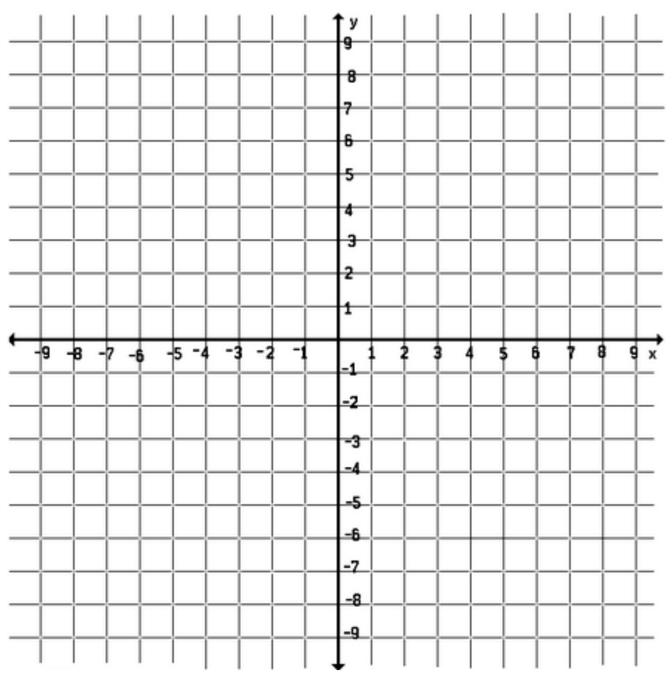
10. ∆ABC: A(-3, 6), B(1, 7) and C(2, 4)
∆A`B`C`: A`(-1, 0), B`(-5, -1) and C`(-6, 2)



11. ∆ABC: A(-1, 1), B(2, 5) and C(5, 1)
∆A`B`C`: A`(3, 3), B`(0, -1) and C`(-3, 3)

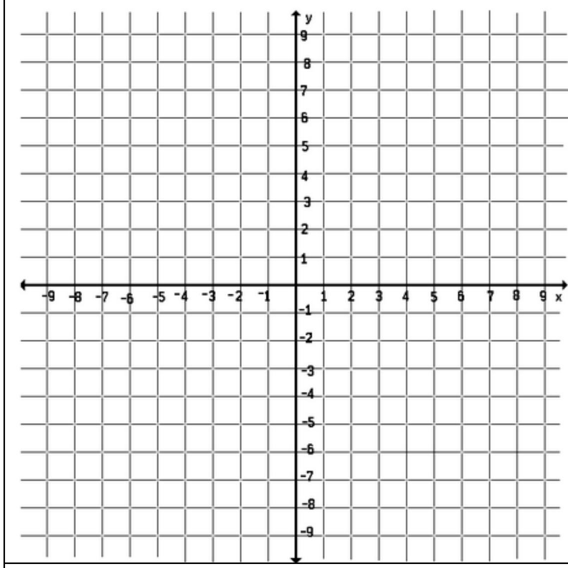


12. ∆ABC: A(3, 8), B(0, 2) and C(5, 4)
∆A`B`C`: A`(1, 4), B`(4, 10) and C`(-1, 8)



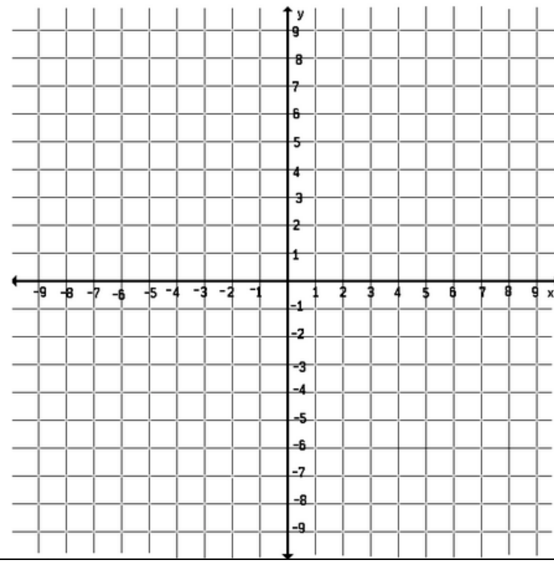
13. The vertices of ΔDEF are D(4, 3), E(8, 1) and F(8, 3). If ΔDEF is reflected through the point (4, 1), find the coordinates of the images ΔD`E`F`

D(4, 3)
E(8, 1)
F(8, 3)



14. The vertices of ΔRST are R(0, 0), S(4, -4) and T(5, -1). If ΔRST is reflected through the point (2, 1), find the coordinates of the images ΔR`S`T`

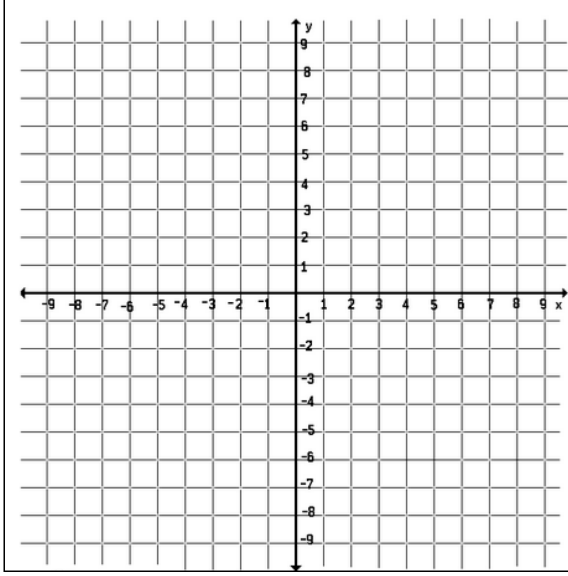
R(0, 0)
S(4, -4)
T(5, -1)



15. The vertices of ΔABC are A(1, 3), B(1, 1) and C(5, 1).

a) give the points of ΔA`B`C` under a point reflection of ΔABC through the origin.

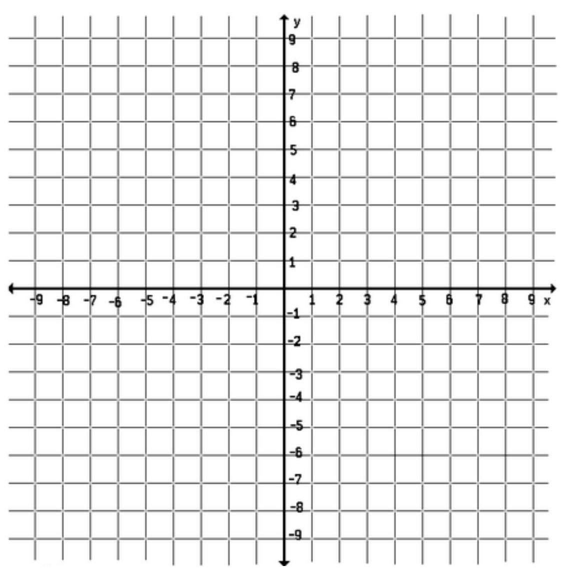
b) give the points of A``B``C``, under a reflection of ΔA`B`C` in the x-axis



16. The vertices of ΔABC are A(1, 3), B(1, 1) and C(5, 1).

a) give the points of ΔA`B`C` under a reflection of ΔABC through the equation x = 1.

b) give the points of A``B``C``, under a reflection of ΔA`B`C` in the y-axis



Translations

A **translation** "slides" an object a fixed distance in a given direction. The original object and its translation have the **same shape and size,** and they **face in the same direction.** A translation creates a figure that is **congruent** with the original figure and preserves distance (length) and orientation (lettering order). A translation is a **direct isometry**. The word "translate" in Latin means "carried across".

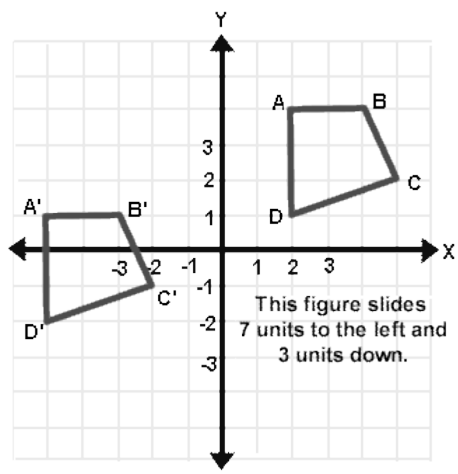
Under a Translation, distance, angle measurement, collinearity and midpoint are preserved.

Translations in the Coordinate Plane

$$T_{a,b}(x, y) = (x+a, y + b)$$

The translation below, moves the figure 7 units to the left and 3 units down.

$$T_{7,-3}(x, y) = (x+7, y-3)$$



Use the rule (x, y) (x + 2, y + 3) to find the image of the given point:			
1. (3, -1)	2. (2, 6)	3. (0, -8)	4. (-5, -3)
Use the rule (x, y) (x - 4, y + 9) to find the image of the given point:			
5. (4, 4)	6. (2, 0)	7. (-3, -10)	8. (-5, 9)
Find the image of the point (2, 7) under the given translation:			
9. $T_{1, 2}$	10. $T_{3, -5}$	11. $T_{-4, 0}$	12. $T_{-2, -6}$

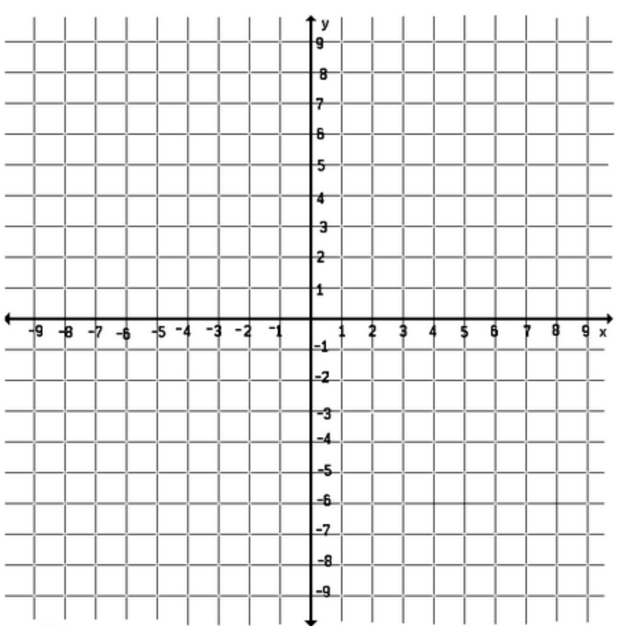
Find the rule for the translation so the image of A is A`		
13. A(3, 8) → A`(4, 6)	14. A(1, 0) → A`(0, 1)	15. A(2, 5) → A`(-1, 1)
16. A(-1, 2) → A`(-2, -3)	17. A(0, -3) → A`(-7, -3)	18. A(4, -7) → A`(4, -2)

19. Given ΔHOT whose vertices are H(-2, 0), O(0, 0) and T(0, 4):

a) Give the coordinates of ΔH`O`T` under the image $T_{2,-3}$ of ΔHOT

b) Give the coordinates of ΔH``O``T`` under the image of $T_{1,2}$ of ΔH`O`T`

c) Name a single transformation that is equivalent to $T_{2,-3}$ followed by $T_{1,2}$

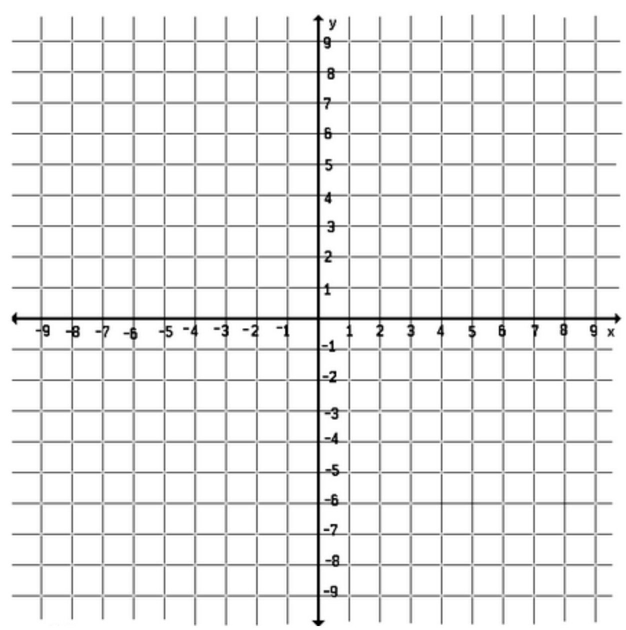


20. Given quadrilateral WARM whose vertices are W(-4, 3), A(0, 1), R(2, 5) and M(0, 5)

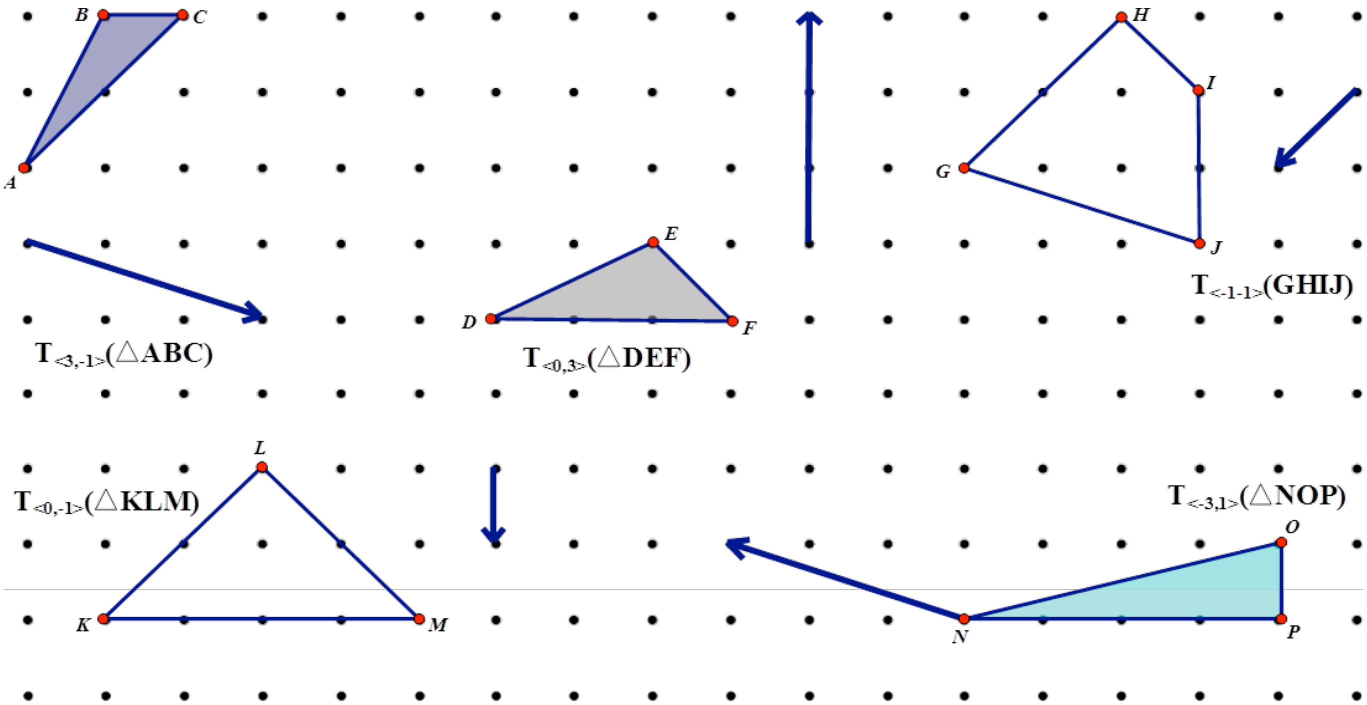
a) Give the coordinates of ΔW`A`R`M`under the image $T_{5,-2}$ of WARM

b) Give the coordinates of ΔW``A``R``M`` under the image of $T_{0,2}$ of ΔW`A`R`M`

c) Name a single transformation that is equivalent to $T_{5,-2}$ followed by $T_{0,2}$

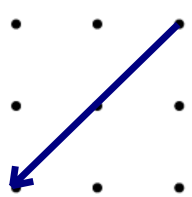


1. Use the grid or patty paper to translate the following figures. Label the image.

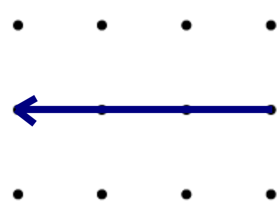


2. Determine the translation coordinate rule from the vector.

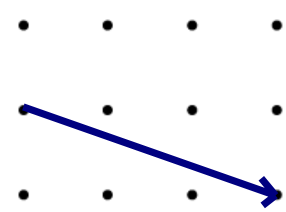
a) T (x,y) ---->
(________,_________)



b) T (x,y) ---->
(________,_________)



c) T (x,y) ---->
(________,_________)



3. Determine the translation rule from the pre-image and image.

a) A (3,5)   A' (-1,3)   T (x,y) ------------- > (____________ , ____________)

b) A (-4,11)   A' (3,0)   T (x,y) ------------- > (____________ , ____________)

c) A (0,-8)   A' (-1,-3)   T (x,y) ------------- > (____________ , ____________)

d) A (8,3)   A' (11,3)   T (x,y) ------------- > (____________ , ____________)

4. Given a translation rule, determine the missing point.

a) T (x,y) ------- > (x + 3, y – 5) A (-4,7) A’ (_______ , ______)

b) T (x,y) ------- > (x -7, y – 1) A (9,1) A’ (_______ , ______)

c) T (x,y) ------- > (x + 1, y + 6) A (_______ , ______) A’ (4,-1)

d) T (x,y) ------- > (x, y + 4) A (8,-4) A’ (_______ , ______)

e) T (x,y) ------- > (x + 3, y + 1) A (_______ , ______) A’ (-4,1)

f) T (x,y) ------- > (x - 8, y - 5) A (_______ , ______) A’ (-3,-3)

5. Convert between vector component form and coordinate form.

a) $T_{<-5,2>}(A) =$ T (x,y) ------- > (______________ , ______________)

b) $T_{<0,-12>}(A) =$ T (x,y) ------- > (______________ , ______________)

c) $T_{<-1.5,-7>}(A) =$ T (x,y) ------- > (______________ , ______________)

6. Write the coordinate rule that matches the description.

a) 4 down and 3 right T (x,y) ------- > (______________ , ______________)

b) left 7 and down 2 T (x,y) ------- > (______________ , ______________)

c) right 1 T (x,y) ------- > (______________ , ______________)

7. What is the resultant translation of Point A after mapping T (x,y) followed by R (x,y).

a) A (-4,8) T (x,y) ------- > (x + 3, y – 7) A’ (_____ , _____) R (x,y) ------- > (x – 8, y – 2) A’’ (_____ , _____)

b) A (2,0) T (x,y) ------- > (x - 1, y) A’ (_____ , _____) R (x,y) ------- > (x - 3, y + 3) A’’ (_____ , _____)

c) A (5,-11) T (x,y) ------- > (x +7, y - 11) A’ (_____ , _____) R (x,y) ------- > (x - 9, y + 9) A’’ (_____ , _____)

8. Can you find a shortcut to doing two translations?

9. What is the pre-image of A’(-5,4) mapped by translation T (x,y) ------ > (x – 5, y + 11)?

Glide Reflection

A **Glide Reflection** is a composition of transformations that consists of a line reflection and a translation in the direction of the line of reflection performed in either order.

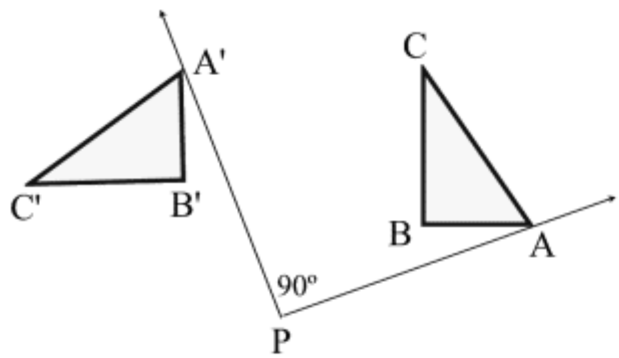
Under a Glide Reflection, distance, angle measurement, collinearity and midpoint are preserved. It is an **isometry** since it preserves distance

Rotations

A **rotation** is a transformation that turns a figure about a fixed point called the **center of rotation**. Rays drawn from the center of rotation to a point and its image form an angle called the **angle of rotation**. A **rotation** is an **isometry**

(notation R_{degrees})

Under a Rotation, distance, angle measurement, collinearity and midpoint are preserved.



Rotations in the Coordinate Plane

Unless stated, the rotation is always counterclockwise.

Rotation of 90°	$R_{90}(x, y) = (-y, x)$
Rotation of 180°	$R_{180}(x, y) = (-x, -y)$
Rotation of 270°	$R_{270}(x, y) = (y, -x)$

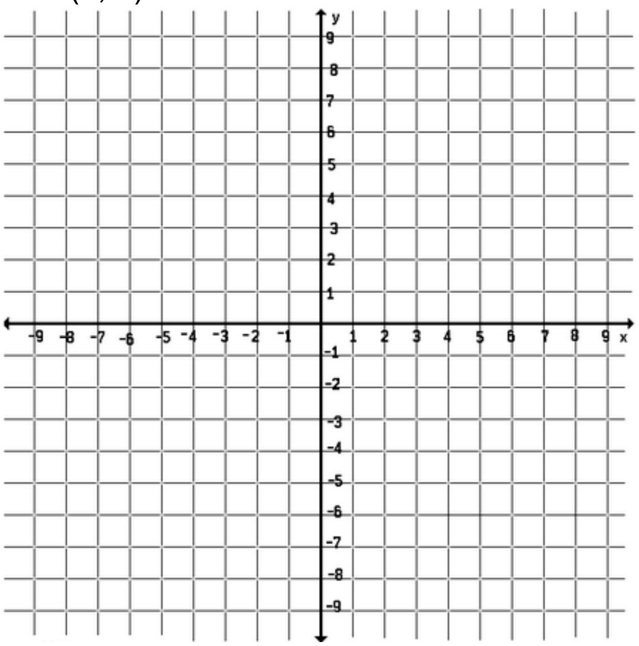
Counting the lunch line.

1. Under a rotation of 90° counterclockwise about the origin, the image of (x, y) is
2. Under a rotation of 180° counterclockwise about the origin, the image of (x, y) is

Using the rule (x, y) $\rightarrow$ (-y, x) and give the coordinates of the image ∆A\`B\`C\` and draw ∆ABC and ∆A\`B\`C

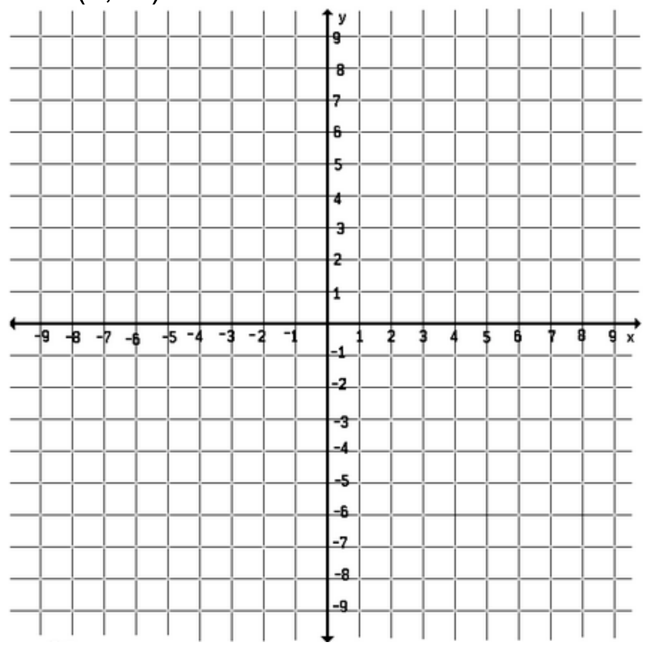
3. ∆ABC: A(1, 1), B(1, 5) and C(4, 5)

(x, y) (-y, x)R_{90}
A(1, 1)
B(1, 5)
C(4, 5)



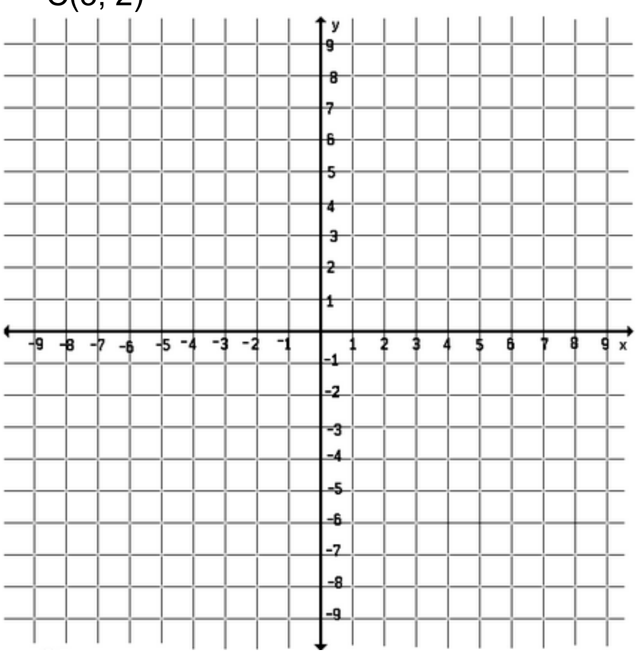
4. ∆ABC: A(4, 3), B(4, -2) and C(1, -1)

(x, y) (-y, x)R_{90}
A(4, 3)
B(4, -2)
C(1, -1)



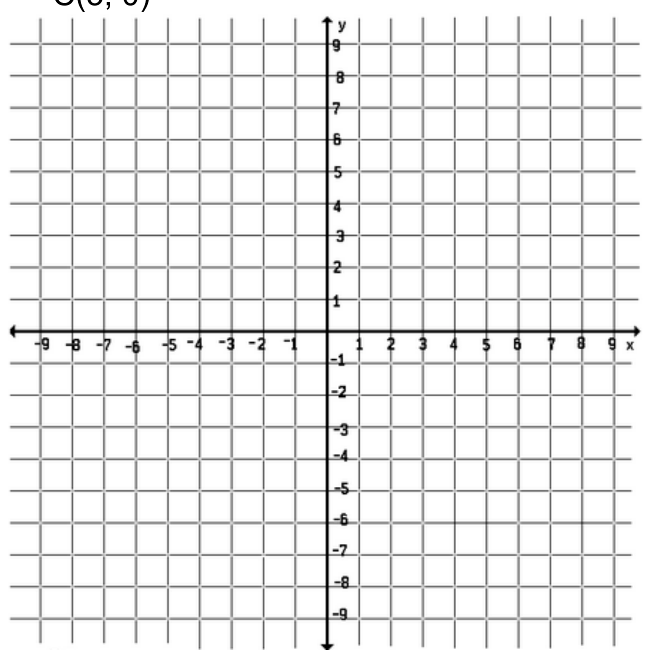
5. ∆ABC: A(1, 2), B(1, 6) and C(3, 2)

(x, y) (-y, x)R_{90}
A(1, 2)
B(1, 6)
C(3, 2)



6. ∆ABC: A(1, 3), B(3, 5) and C(3, 0)

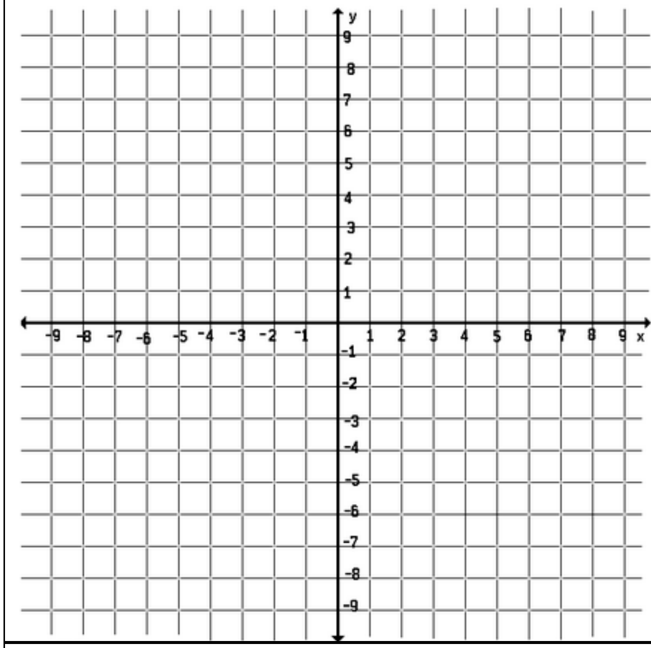
(x, y) (-y, x)R_{90}
A(1, 3)
B(3, 5)
C(3, 0)



7. ΔABC: A(2, 1), B(3, -1) and C(-2, -1)

(x, y) (-y, x)R_{90}

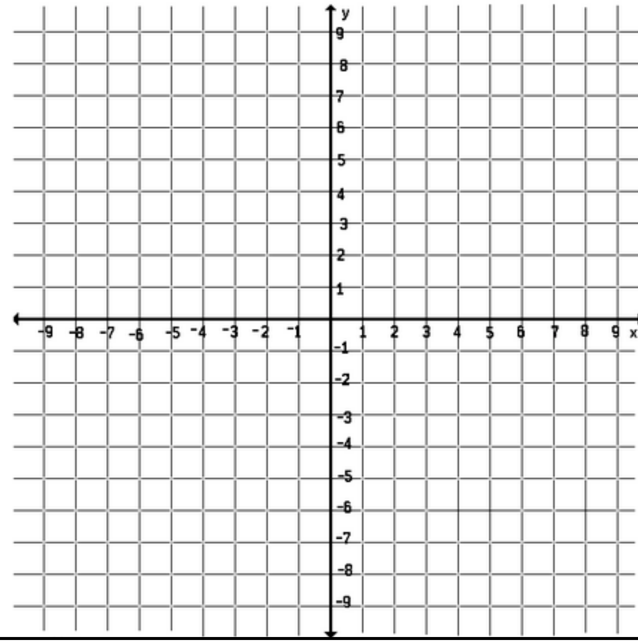
A(2, 1)
B(3, -1)
C(-2, -1)



8. ΔABC: A(0, 0), B(2, 1) and C(-2, 4)

(x, y) (-y, x)R_{90}

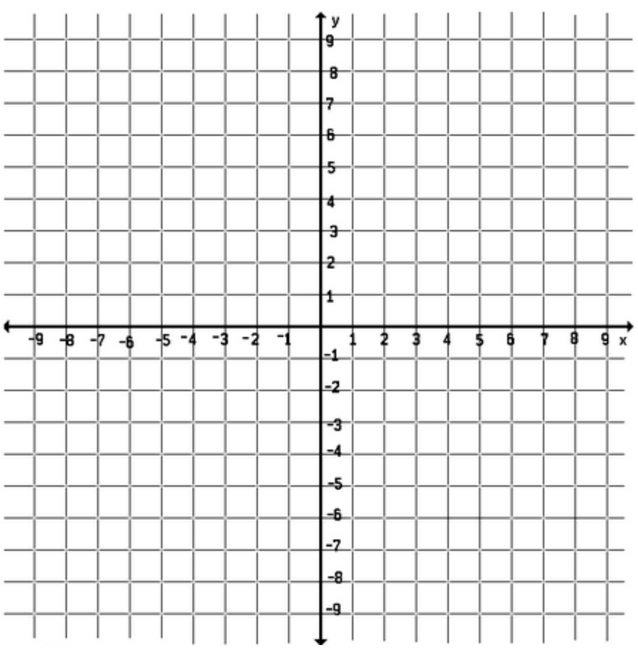
A(0, 0)
B(2, 1)
C(-2, 4)



9. ΔABC: A(-2, 2), B(0, -4) and C(4, -4)

(x, y) (-y, x)R_{90}

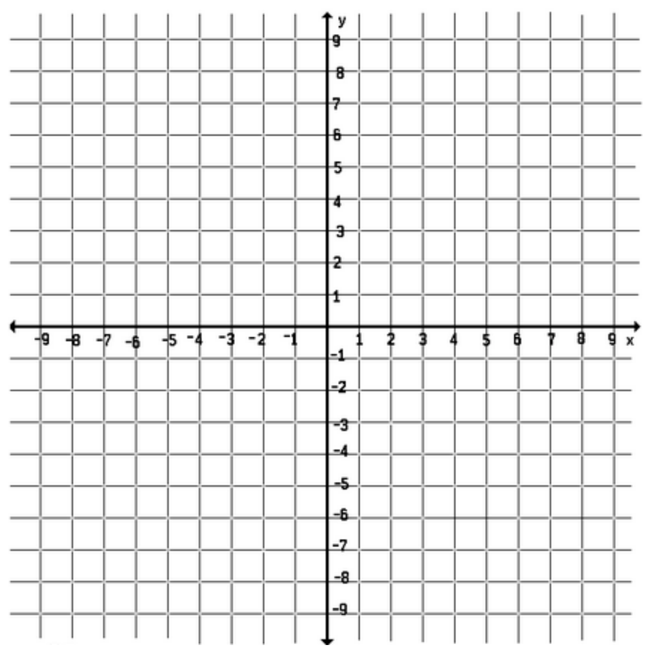
A(-2, 2)
B(0, -4)
C(4, -4)



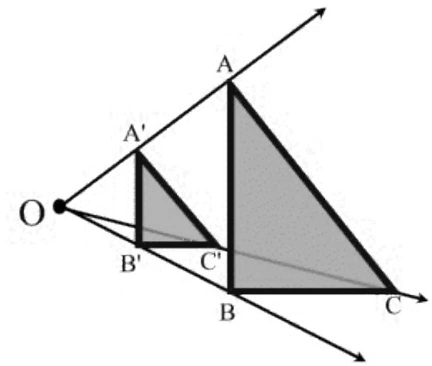
10. ΔABC: A(2, 5), B(0, 1) and C(-3, 7)

(x, y) (-y, x)R_{90}

A(2, 5)
B(0, 1)
C(-3, 7)



Dilations in the Coordinate Plane



$$D_k(x, y) = (kx, ky)$$

Under a dilation, distance is **not** preserved.

Using the rule (x, y) → (4x, 4y), find the image of the given point:			
1. (3, -1)	2. (2, 6)	3. (0, -8)	4. (-5, -3)
Find the image of the given point under a dilation of 5			
5. (5, 7)	6. (6, 2)	7. (-1, -4)	8. (3, 0)
Under D_{-3} find the image of the given point			
9. (5, 7)	10. (-3, 8)	11. (0, -2)	12. (6, 6)

Write a single rule for a dilation by which the image of A is A`		
13. A(2, 5) → A`(4, 10)	14. A(3, -1) → A`(21, -7)	15. A(10, 4) → A`(5, 2)
16. A(-20, 8) → A`(-5, 2)	17. A(4, 6) → A`(6, 9)	18. A(4, -3) → A`(-8, 16)
19. A(-2, 5) → A`(8, -20)	20. A(-12, 9) → A`(-8, 6)	21. A(0, 9) → A`(0, -3)

NOTATION CONSISTENCY

REFLECTION

$R_{x\ axis}$ Reflection over the x axis R_x is probably okay as well

$R_{y\ axis}$ Reflection over the y axis R_y is probably okay as well

$R_{x=3}$ Reflection over the x = 3 line

$R_{y=x}$ Reflection over the y = 1x line

R_m Reflection over line m

$R_{\overline{AB}}$ Reflection over segment AB

$R_{\overleftrightarrow{AB}}$ Reflection over line AB

A Reflection is recognizable because it will have only ONE item as a subscript… the line of reflection.
(Some use a small r for reflection and a capital R for rotation.)

ROTATION

$R_{O,89°}$ Rotation about Point O for a positive 89°

When O is used it is implied that O = Origin at (0, 0)

$R_{P,-134°}$ Rotation about Point P for a negative 134°

$R_{(2,3),42°}$ Rotation about location (2,3) for a positive 42°

A rotation is recognizable because it will have TWO items in the subscript… a center and a degree.

TRANSLATION

$T_{\langle -6,4\rangle}$ Translate 6 left and 4 up

A translation is recognizable because it will have vector notation.

DILATION

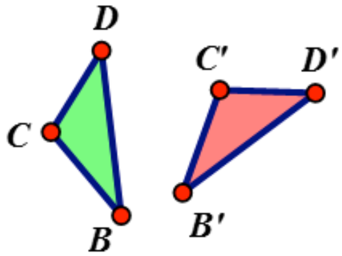
$D_{O,3}$ Dilation from point O a scale factor of 3

$D_{O,\frac{1}{2}}$ Dilation from point O a scale factor of 1/2

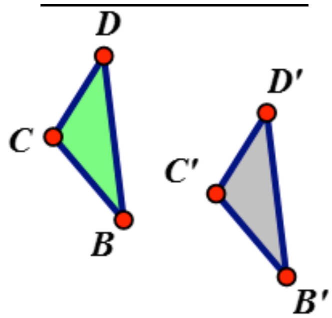
$D_{A,-2}$ Dilation from point A a scale factor of -2

1. – 4. Which transformation has taken place?

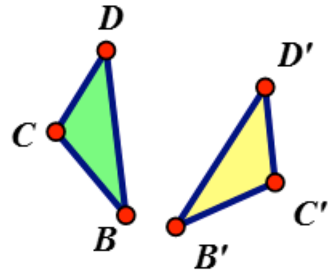
1. ______________



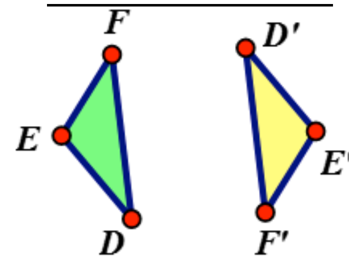
2. ______________



3. ______________



4. ______________

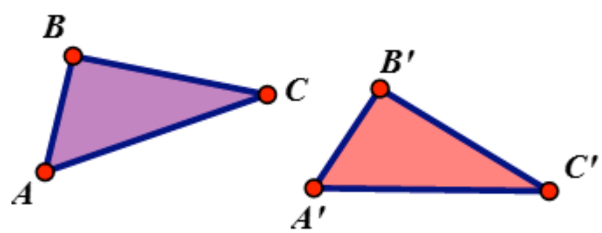


2. Complete the chart.

Relationship between pre-image and image	ROTATION	REFLECTION	TRANSLATION
Distances	SAME OR DIFFERENT	SAME OR DIFFERENT	SAME OR DIFFERENT
Orientation	SAME OR DIFFERENT	SAME OR DIFFERENT	SAME OR DIFFERENT

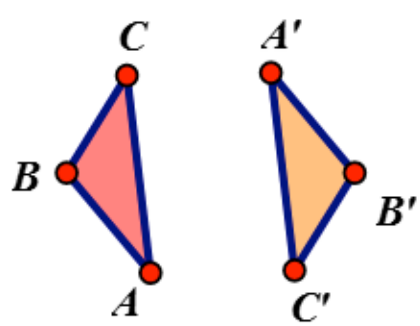
3. Given that ΔABC was mapped to ΔA'B'C' using a single transformation.
a) Why couldn't this mapping have resulted by a single translation?

b) What transformation must have mapped these two triangles?
Explain your answer



4. Given that ΔABC was mapped to ΔA'B'C' using a single transformation.
a) Why couldn't this mapping have resulted by a single reflection?

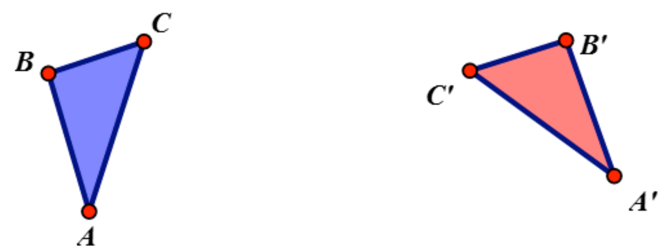
b) What transformation must have mapped these two triangles?
Explain your answer.



5. ΔABC is congruent to $\Delta A'B'C'$. A student tries to determine which of these single transformations mapped ΔABC onto $\Delta A'B'C'$. She concludes that a reflection had to be involved and more than one transformation had to map these on two triangles.

a) How can she conclude that a reflection was involved?

b) How can she conclude that this wasn't just a single reflection?



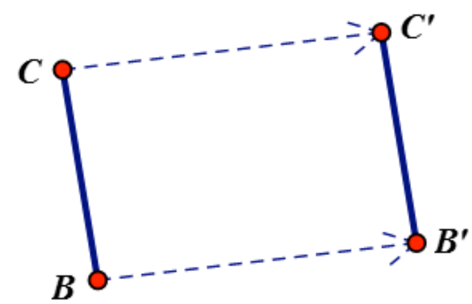
6 . $\overline{BC}$ was translated by the arrow making $\overline{BC} \cong \overline{B'C'}$ and $\overline{BC} \parallel \overline{B'C'}$.

a) What other segments in the diagram are congruent?

____

b) What other segments in the diagram are parallel?

____



1. ROTATION BY 90° ABOUT THE ORIGIN (Use Patty Paper to help you with these!!)

PRE-IMAGE LOCATION	REFLECTION	IMAGE LOCATION
a) Place ΔDEF on the coordinate grid at D(0,4), E(3,4) and F(0,0)	$R_{O,90°}$	D'(___, ___), E'(___, ___), & F'(___, ___)
b) Place ΔABC on the coordinate grid at A(-3,6), B(-1,1) and C(-5,1)	$R_{O,90°}$	A'(___, ___), B'(___, ___), & C'(___, ___)
c) Place ΔMNP on the coordinate grid at M(-6,5), N(1,2) and P(-4,2)	$R_{O,90°}$	M'(___, ___), N'(___, ___), & P'(___, ___)

Create a transformation rule for rotation about the origin, 90°. $R_{O,90°}(x, y) = (\_\_\_\_\_\_, \_\_\_\_\_\_)$

2. ROTATION BY 180° ABOUT THE ORIGIN (Use Patty Paper to help you with these!!)

PRE-IMAGE LOCATION	REFLECTION	IMAGE LOCATION
a) Place ΔDEF on the coordinate grid at D(0,4), E(3,4) and F(0,0)	$R_{O,180°}$	D'(___, ___), E'(___, ___), & F'(___, ___)
b) Place ΔABC on the coordinate grid at A(5,6), B(7,1) and C(3,1)	$R_{O,180°}$	A'(___, ___), B'(___, ___), & C'(___, ___)
c) Place ΔMNP on the coordinate grid at M(1,4), N(8,1) and P(3,1)	$R_{O,180°}$	M'(___, ___), N'(___, ___), & P'(___, ___)

Create a transformation rule for rotation about the origin, 180°. $R_{O,180°}(x, y) = (\_\_\_\_\_\_, \_\_\_\_\_\_)$

3. ROTATION BY 270° (-90°) ABOUT THE ORIGIN (Use Patty Paper to help you with these!!)

PRE-IMAGE LOCATION	REFLECTION	IMAGE LOCATION
a) Place ΔDEF on the coordinate grid at D(0,4), E(3,4) and F(0,0)	$R_{O,270°}$	D'(___, ___), E'(___, ___), & F'(___, ___)
b) Place ΔABC on the coordinate grid at A(-3,6), B(-1,1) and C(-5,1)	$R_{O,270°}$	A'(___, ___), B'(___, ___), & C'(___, ___)
c) Place ΔMNP on the coordinate grid at M(-3,5), N(-6,-2) and P(-6,3)	$R_{O,270°}$	M'(___, ___), N'(___, ___), & P'(___, ___)

Create a transformation rule for rotation about the origin, 270° $R_{O,270°}(x, y) = (\_\_\_\_\_\_, \_\_\_\_\_\_)$

4. Using your rules above, determine the location of the missing location.

a) $R_{O,90°}(-6,4) = (\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_)$

b) $R_{x=0}(-3,-5) = (\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_)$

c) $R_{O,270°}(\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_) = (-3,-4)$

d) $R_{x\ axis}(7,-8) = (\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_)$

e) $R_{y\ axis}(\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_) = (8,4)$

f) $R_{O,-270°}(-5,-8) = (\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_)$

g) $R_{y=0}(2,8) = (\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_)$

h) $R_{O,180°}(\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_) = (0,9)$

i) $R_{x\ axis}(\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_) = (3,-5)$

j) $R_{O,180°}(-4,2) = (\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_)$

k) $R_{y\ axis}(\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_) = (-4,2)$

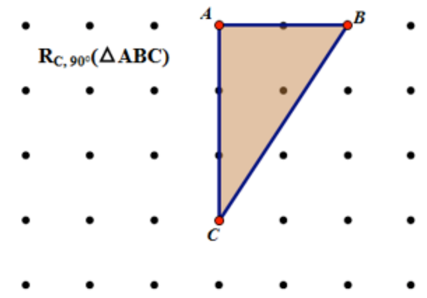
l) $R_{y\ axis}(-2,15) = (\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_)$

m) $R_{O,90°}(\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_) = (9,-3)$

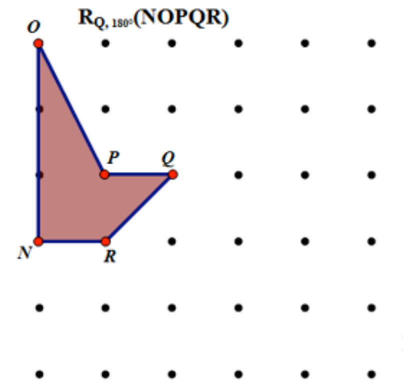
n) $R_{O,-90°}(10,3) = (\_\_\_\_\_\_\_\_, \_\_\_\_\_\_\_\_)$

1. – 6. Use the grid to rotate the following figures. Label the image.

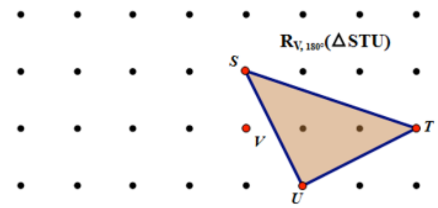
1.



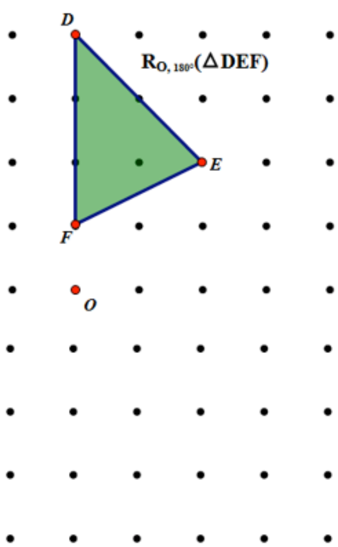
2.



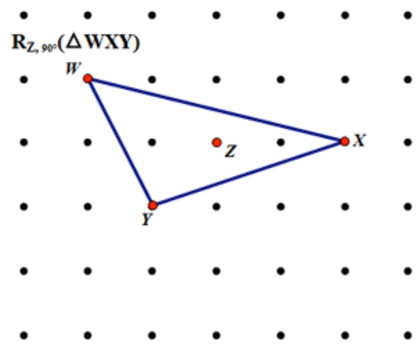
3.



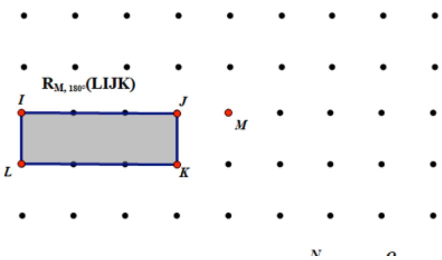
4.



5.

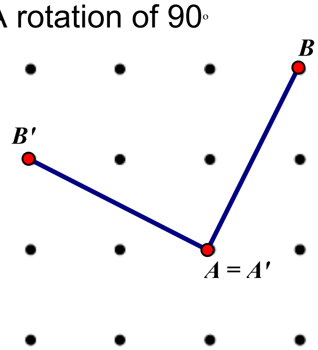


6.

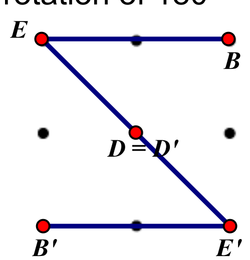


7. – 9. Circle the center of rotation for the following pre-image and images.

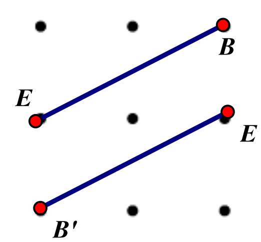
7. A rotation of 90°



8. A rotation of 180°



9. A rotation of 180°



10. Determine the pre-image coordinates, then rotate it, and determine the image coordinates.

$R_{O,90°}(\Delta ABC)$

a) A = (____ , ____) $R_{O,90°}(\Delta ABC)$ A ' = (____ , ____)

b) B = (____ , ____) $R_{O,90°}(\Delta ABC)$ B ' = (____ , ____)

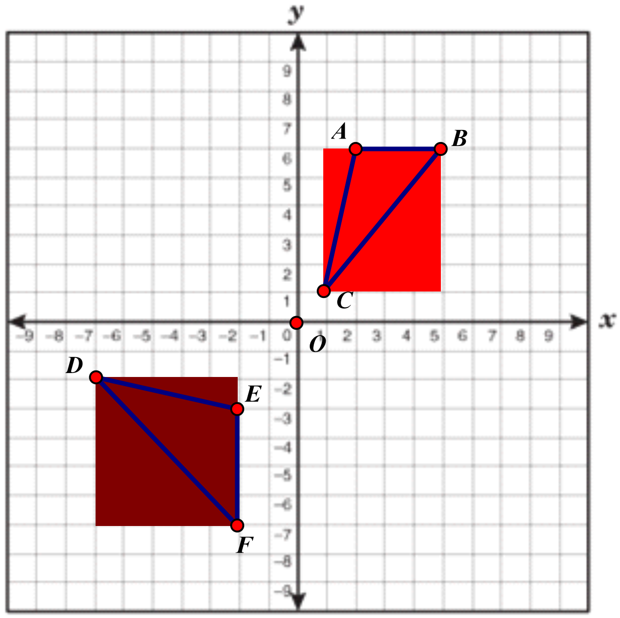
c) C = (____ , ____) $R_{O,90°}(\Delta ABC)$ C ' = (____ , ____)

$R_{O,90°}(\Delta DFE)$

d) D = (____ , ____) $R_{O,90°}(\Delta DFE)$ D ' = (____ , ____)

e) E = (____ , ____) $R_{O,90°}(\Delta DFE)$ E ' = (____ , ____)

f) F = (____ , ____) $R_{O,90°}(\Delta DFE)$ F ' = (____ , ____)



11. Determine the name of the point that meets the given conditions.

a) $R_{G,60°}(A)$ = ________

b) $R_{G,180°}(B)$ = ________

c) $R_{G,300°}(D)$ = ________

d) $R_{G,-120°}$(________) = B

e) $R_{G,240°}(E)$ = ________

f) $R_{G,-240°}(F)$ = ________

g) $R_{A,60°}(B)$ = ________

h) $R_{C,120°}(D)$ = ________